OMORAGBON OSAFURE FAVOUR
19|MHS01|344
MEDICINE AND SURGERY.

Question
Integrate the following
a $2x^2 \ln x$
b $3t\, e^{2t}$
c $x^2 \sin x$
d $\cos 5x \cos 6x$
e $\sin 7x \cos 2x$

Solution
a $\int 2x^2 \ln x \, dx$

$U = \ln x \qquad dv = 2x^2$

$du = \frac{dx}{x} \qquad v = \frac{2x^3}{3}$

using $\int U dv = UV - \int V du$

$$\int 2x^2 \ln x dx = \ln x \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$$

$$\int 2x^2 \ln x dx = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^2}{3} dx$$

$$\int 2x^2 \ln x dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$$

$$\int 2x^2 \ln x dx = \frac{2x^3}{3} \left[\ln x - \frac{1}{3}\right] + C$$

b $\int 3t e^{2t}\,dt$

$U = 3t$ $\qquad dv = e^{2t}$

$du = 3\,dt$ $\qquad v = \frac{1}{2}\cdot e^{2t}$

Using $\int U\,dv = Uv - \int v\,dU$

$\int 3te^{2t}\,dt = 3t\cdot\frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t}\cdot 3\,dt$

$= \frac{3}{2}te^{2t} - \int \frac{3}{2}e^{2t}\,dt$

$\int te^{2t}\,dt = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$

c $\int x^2 \sin x\,dx$

$U = x^2$ $\qquad dv = \sin x$

$du = 2x\,dx$ $\qquad v = -\cos x$

Using $\int U\,dv = Uv - \int v\,dU$

$\int x^2 \sin x\,dx = x^2 - \cos x - \int -\cos x\, 2x\,dx$

$\int -\cos x\, 2x\,dx = 2x\sin x - \int 2 \sin x\,dx$

$[du = 2dx \qquad v = \sin x]$

$\int -\cos x\, 2x\,dx = 2x\sin x - (-2\cos x)$

$= 2x\sin x + 2\cos x$

$\int x^2 \sin x\,dx = x^2 - \cos x - 2x\sin x \mp 2\cos x$

$= x^2 - 2x\sin x \mp 3\cos x + C$

$\int x^2 \sin x\,dx = x^2 - 2x\sin x - 3\cos x + C$

d. $\int \cos 5x \cos 6x \, dx = \frac{1}{2}$

Using $\frac{1}{2}[\cos[A+B] + \cos[A-B]]$

$$\int \cos 5x \cos 6x \, dx = \int \frac{1}{2}[\cos 11x + \cos -1x] \, dx$$

$$= \frac{1}{2}\left[\frac{1}{11}\sin 11x - \sin x\right] dx$$

$$= \frac{1}{22}\sin 11x - \frac{1}{2}\sin x + c$$

e. $\int \sin 7x \cos 2x \, dx$

Using $\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int [\sin 9x + \sin 5x] \, dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\frac{-1}{9}\cos 9x - \frac{1}{5}\cos 5x\right] + c$$

$$\int \sin 7x \cos 2x \, dx = \frac{-1}{18}\cos 9x - \frac{1}{10}\cos 5x + c$$