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19/MHS01/440
Medicine and Surgery.
MAT 104.

Integrate the following

1. $\int 2x^2 \ln x$.

Solution

$\int u dv = uv - \int v du$.

$u = \ln x \qquad du = \frac{1}{x} dx$.

$dv = 2x^2 \quad , \quad v = \frac{2x^3}{3}$.

$\ln x \frac{2}{3} x^3 - \int \frac{2x^{\cancel{3}}}{3} \frac{1}{\cancel{x}} dx$

$\ln x \frac{2}{3} x^3 - \int \frac{2x^2}{3} dx$

$\ln x \frac{2}{3} x^3 - \frac{2}{3} \left(\frac{x^3}{3} \right) + c$.

$\int 2x^2 \ln x = \frac{2}{3} x^3 \left(\ln x - \frac{1}{3} \right) + c$

2.) $\int 3te^{2t} dt$.

$u = 3t \qquad du = 3\,dt$

$dv = e^{2t} \qquad v = \frac{1}{2} e^{2t}$

$\int u dv = uv - \int v du$

$3t \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} . 3\,dt$.

$\frac{3te^{2t}}{2} - \int \frac{3}{2} e^{2t}$

$\frac{3}{2} te^{2t} - \left(\frac{1}{2} \times \frac{3}{2} e^{2t} \right)$.

$= \frac{3}{2} te^{2t} - \frac{3}{4} e^{2t}$

3. $\int x^2 \sin x \, dx.$

$u = x^2 \qquad du = 2x\,dx$

$dv = \sin x \qquad v = -\cos x.$

$uv - \int v\,du$

$x^2 - \cos x - \int -\cos x \, 2x\,dx$

$u = 2x \qquad du = 2$

$dv = -\cos x \qquad v = \sin x$

$-x^2 \cos x - [uv - \int v\,du].$

$-x^2 \cos x - [2x \sin x - \int 2 \sin x.]$

$-x^2 \cos x - 2x \sin x - 2(-\cos x) + c$

$-x^2 \cos x - 2x \sin x + 2 \cos x + c.$

4) $\cos 5x \cos 6x.$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\cos 5x \cos 6x = \frac{1}{2}[\cos(5x+6x) + \cos(5x-6x)]$

$= \frac{1}{2}[\cos(11x) + \cos(-x)]$

$\int \cos 5x \cos 6x \, dx = \int \frac{1}{2}[\cos(11x) + \cos(-x)]\,dx.$

$= \frac{1}{2} \int \cos(11x) + \cos(-x)\,dx$

$= \frac{1}{2}\left[\sin(11x) \cdot \frac{1}{11} + \sin(-x) \cdot \frac{1}{-1}\right.$

$= \cancel{\frac{1}{2}}\left[\frac{\sin 11x}{22} + \frac{\sin x}{2}\right] + c$

$\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + c.$

5.) $\int \sin 7x \cos 2x \, dx$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\sin 7x \cos 2x = \frac{1}{2}[\sin(7x+2x) + \sin(7x-2x)]$

$$= \frac{1}{2}\left[\sin(9x) + \sin(5x)\right]$$

$$\int \sin 7x \cos 2x \, dx = \int \frac{1}{2} \sin(9x) + \sin(5x)) \, dx.$$

$$= \frac{1}{2} \int \sin(9x) + \sin(5x) \, dx$$

$$= \frac{1}{2}\left[-\cos(9x) \cdot \frac{1}{9} + -\cos(5x) \times \frac{1}{5}\right]$$

$$= \frac{1}{2}\left[\frac{-\cos(9x)}{9} + \frac{-\cos(5x)}{5}\right]$$

$$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$