20TH MAY, 2020.

AKPOFURE TESE
19/MHS01/077
100 LEVEL
MEDICINE AND SURGERY
MEDICINE AND HEALTH SCIENCES
MAT 104 - GENERAL MATHEMATICS III (CALCULUS)

Integrate the following functions

1. $2x^2 \ln x$

Solution

$2x^2 \ln x \, dx$

Let: $u = \ln x$ $\qquad dv = 2x^2$

$du = \frac{1}{x} dx \qquad v = \frac{2x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$\int u\,dv = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^{\cancel{3}2}}{3} \cdot \frac{dx}{\cancel{x}}$

$\int u\,dv = \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$

$\int u\,dv = \frac{2x^3}{3} \ln x - \frac{2x^3}{3 \times 3} + c$

$\therefore \int 2x^2 \ln x \, dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c$ OR $\frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + c$

2. $3te^{2t}$

Solution

$3te^{2t}\,dt$

Let: $u = 3t$ $\qquad dv = e^{2t}$

$du = 3\,dt \qquad v = \frac{1}{2} e^{2t}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dt$

$\int 3te^{2t} = \frac{3t}{2}e^{2t} - \int \frac{3}{2}e^{2t} \cdot dt$

$\int 3te^{2t} = \frac{3te^{2t}}{2} - \frac{1}{2} \cdot \frac{3}{2}e^{2t} + c$

$\therefore \int 3te^{2t}dt = \left(\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}\right) + c$

3. $x^2 \sin x$

Solution

$x^2 \sin x\,dx$

$u = x^2 \qquad dv = \sin x$

$\frac{du}{dx} = 2x \qquad v = -\cos x$

$du = 2x\,dx$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x\,dx = x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$

$\int x^2 \sin x\,dx = -x^2\cos x + \left(\int 2x \cos x\,dx\right)$ — *

Solving $\left(\int 2x \cos x\,dx\right)$

$\int u\,dv = uv - \int v\,du$

$u = 2x \qquad dv = \cos x$

$du = 2dx \qquad v = \sin x$

$\therefore \int 2x\cos x\,dx = 2x\sin x - \int \sin x \cdot 2dx$

$\int 2x\cos x\,dx = 2x\sin x - \int 2\sin x\,dx$

$\int 2x\cos x\,dx = 2x\sin x + 2\cos x + c$

Putting the above in *

$\int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + c$

4. $\cos 5x \cos 6x$

Solution

$\cos 5x \cos 6x \, dx$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$

Let $A = 5x$ and $B = 6x$

$\cos 5x \cos 6x \, dx = \frac{1}{2}(\cos(5x+6x) - \cos(5x-6x))$

$\cos 5x \cos 6x \, dx = \frac{1}{2}\left(\cos 11x - (-\cos x)\right)$

$\cos 5x \cos 6x \, dx = \frac{1}{2}\int(\cos 11x + \cos x)$

$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\left(\frac{\sin 11x}{11} + \sin x\right) + c$

$\therefore \int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + c$ //

5. $\sin 7x \cos 2x$

Solution

$\sin 7x \cos 2x \, dx$

$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

Let $A = 7x$ and $B = 2x$

$\sin 7x \cos 2x \, dx = \frac{1}{2}(\sin(7x+2x) + \sin(7x-2x))$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}(\sin 9x + \sin 5x)$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right] + c$

$\therefore \int \sin 7x \cos 2x \, dx = \frac{-\cos 9x}{9} - \frac{\cos 5x}{5} + c$ //