1 $\int 2x^2 \ln x$

$u = \ln x$; $dv = 2x^2$

$du/dx = \frac{1}{x}$, $v = \frac{2}{3}x^3$

$dx = x$

$\int 2x^2 \ln x = \frac{2}{3}x^3 \ln x - \int \frac{2}{3}x^3 \cdot \frac{1}{x}\,dx$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^2\,dx$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\left(\frac{x^3}{3}\right) + c$

$= \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + c$

2. $\int 3t\,e^{2t}$ $\int 3te^{2t}$

$u = 3t$, $dv = e^{2t}$

$\frac{du}{dt} = 3$, $v = \frac{1}{2}e^{2t}$

$\int 3te^{2t}$

$= \frac{1}{2}e^{2t} 3t - \int \frac{3}{2}e^{2t}$

$= \frac{3t}{2}e^{2t} - \frac{3}{2}\int e^{2t}$

$= \frac{3}{2}te^{2t} - \frac{3}{2}\cdot\frac{1}{2}e^{2t} + c$

$= \frac{3}{2}e^{2t}\left[t - \frac{1}{2}\right] + c$

3. $\int x^2 \sin x$

$u = x^2$, $dv = \sin x$

$du/dx = 2x$, $v = -\cos x$

$\int x^2 \sin x = -x^2\cos x - \int -2x\cos x$

$= -x^2\cos x + 2\int x\cos x$

$\int x\cos x$ .... ①

$u = x$, $dv = \cos x$

$\frac{du}{dx} = 1$, $v = \sin x$

$\int x\cos x = x\sin x - \int \sin x$

$= x\sin x - (-\cos x) + c$

②.... $= x\sin x + \cos x + c$

Put ② into ①

$\int x^2 \sin x = -x^2\cos x + 2[\sin x + \cos x] + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + c$

$= 2x\sin x + 2\cos x - x^2\cos x + c$

4 $\int \cos 5x \cos 6x$

$A = 5x ; \quad B = 6x$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\cos 5x \cos 6x$

$= \frac{1}{2}[\cos(11x) + \cos(-x)]$

$\int \cos 5x \cos 6x$

$= \int \frac{1}{2}[\cos 11x + \cos(-x)]$

$= \frac{1}{2}\int \cos 11x + \cos(-x)$

$= \frac{1}{2}\left[\frac{1}{11} \cdot \sin 11x + \int -1 \sin x\right]$

$= \frac{1}{2}\left[\frac{\sin 11x}{11} + \sin(-x)\right]$

$= \frac{\sin 11x}{22} + \frac{\sin(-x)}{2}$

(5) $\sin 7x \cos 2x$

$A = 7x , \quad B = 2x$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$A+B = 9x ; \quad A-B = 5x$

$\sin 7x \cos 2x = \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos 2x$

$= \int \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{1}{2}[\sin 9x + \sin 5x)$

$= \frac{1}{2}\left[-\frac{1}{9}\cos 9x - \frac{1}{5}\cos 5x\right] + C$

$= \frac{1}{2}\left[-\frac{1}{9}\cos 9x - \frac{1}{5}\cos 5x\right] + C$

$= -\frac{1}{18}\cos 9x - \frac{1}{10}\cos 5x + C$