

19/11/2016

(1)  $\int 2x^2 \ln x \, dx$

$$\int 2x^2 \ln x \, dx = 2 \int x^2 \ln x \, dx$$

using integration by part

$$uv = \int u \, dv + \int v \, du \quad \text{let } 2 \ln x = u \quad \text{and } dv = x^2$$

$$v = \int dv = \int x^2 \, dx \quad ; \quad \frac{du}{dx} = \frac{2}{x}$$

$$v = \frac{x^3}{3}$$

$$\therefore 2 \ln x \cdot \frac{x^3}{3} = \int 2 \ln x \cdot x^2 \, dx + \int \frac{x^3}{3} \cdot \frac{2}{x} \, dx$$

rearranging

$$\int 2x^2 \ln x \, dx = \frac{2x^3 \ln x}{3} - \int \frac{2x^2}{3} \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \int x^2 \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \left( \frac{x^3}{3} \right)$$

$$\therefore \int 2x^2 \ln x = \frac{2}{3} \left( x^3 \ln x - \frac{x^3}{3} \right) + C$$

②

$$\int 3te^{2t} dt$$

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$$\begin{aligned} \text{let } u &= 3t \\ du &= 3 \\ dv &= e^{2t} \\ v &= \frac{e^{2t}}{2} \end{aligned}$$

$$UV = \int u dv + \int v du$$

$$\frac{3te^{2t}}{2} = \int 3te^{2t} + \int \frac{3e^{2t}}{2}$$

$$\int 3te^{2t} = \frac{3te^{2t}}{2} - \int \frac{3}{2} e^{2t}$$

$$\int 3te^{2t} = \frac{3te^{2t}}{2} - \frac{3}{2} \int e^{2t}$$

$$= \int 3te^{2t} = \frac{3te^{2t}}{2} - \frac{3}{2} \left( \frac{e^{2t}}{2} \right)$$

$$\int 3te^{2t} = \frac{3}{2} \left( te^{2t} - \frac{e^{2t}}{2} \right)$$


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⑤

$$\int x^2 \sin x dx$$

$$\begin{aligned} u &= x^2 \\ du &= 2x \\ dv &= \sin x \\ v &= -\cos x \end{aligned}$$

$$\int u dv = UV - \int v du$$

$$\int x^2 \sin x = -x^2 \cos x - \int (-\cos x) \cdot 2x$$

$$\int x^2 \sin x = -x^2 \cos x + \int 2x \cos x \quad \text{--- (1)}$$

$$\int 2x \cos x \quad \text{let } u = 2x, \quad du = 2$$

$$dv = \cos x, \quad v = \sin x$$

$$\int u dv = UV - \int v du \quad ; \quad \int 2x \cos x = 2x \sin x + \int 2 \sin x$$

$$\int 2x \cos x = 2x \sin x - 2 \int \sin x$$

$$\int 2x \cos x = 2x \sin x + 2 \cos x \quad \text{--- (1)}$$

~~Substituting~~ Sub. The value of  $\int 2x \cos x$  into eq. (1)

$$\int x^2 \sin x = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$


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$$\int \cos 5x \cos 6x \, dx$$

$$\int \cos 5x \cos 6x = \int \cos A \cos B$$

$$\int \cos A \cos B = \frac{1}{2} \left( \cos(A-B) + \cos(A+B) \right)$$

$$\cos A \cos B = \frac{1}{2} \left( \cos(A-B) + \cos(A+B) \right)$$

$$\therefore \int \cos A \cos B = \int \frac{1}{2} (\cos(A-B) + \cos(A+B))$$

~~$\int \cos 5x \cos 6x$~~  let  $A = 5x$  and  $B = 6x$

$$\int \cos 5x \cos 6x = \frac{1}{2} (\cos(5x-6x) + \cos(5x+6x))$$

$$\int \cos 5x \cos 6x = \frac{1}{2} (\cos(-x) + \cos(11x))$$

$$\int \cos 5x \cos 6x = \frac{1}{2} (\cos x + \cos 11x)$$

$$\int \cos 5x \cos 6x = \frac{1}{2} \left( \sin x + \frac{\sin 11x}{11} \right) + C$$

$$\therefore \int \cos 5x \cos 6x = \frac{\sin x}{2} + \frac{\sin 11x}{22} + C$$


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⑤  $\int \sin 7x \cos 2x \, dx$

$\sin 7x \cos 2x$ , let  $A = 7x$  and  $B = 2x$

$$\sin A \cos B = \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

$$\int \sin A \cos B = \int \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \int \sin(7x+2x) + \sin(7x-2x)$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \int \sin 9x + \sin 5x$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \left( \int \sin 9x + \int \sin 5x \right)$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \left( -\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right) + C$$

$$\int \sin 7x \cos 2x = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$