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1. Integrate the following functions:

a) $2x^2 \ln x \, dx$

$U = \ln x \; ; \; dv = 2x^2$

$du = 1/x \, dx \; ; \; V = \frac{2x^3}{3}$

Recall that:

$\int U dv = Uv - \int V du$

$\Rightarrow = \left(\ln x \cdot \frac{2x^3}{3}\right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

$= \left(\ln x \cdot \frac{2x^3}{3}\right) - \int \frac{2x^2}{3} dx$

$= \left(\ln x \cdot \frac{2x^3}{3}\right) - \frac{2x^3}{9} + C$

$= \frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$

b) $3te^{2t} dt$

$U = 3t \; ; \; dv = e^{2t}$

$du = 3dx \; ; \; V = \cancel{\frac{1}{2}}e^{2t} = 1/2 e^{2t}$

$\int U dv = Uv - \int V du$

$= 3t \cdot \frac{1e^{2t}}{2} - \int \frac{1e^{2t}}{2} \cdot 3$

$\Rightarrow 3t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3$

$= \frac{3te^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3$

$= \frac{3te^{2t}}{2} - \int \frac{3e^{2t}}{2}$

$\Rightarrow \frac{3te^{2t}}{2} - \left[ \frac{3}{2} \int e^{2t} \right]$

Let $u = 2t$

$\frac{du}{dt} = 2$ ; $\frac{dt}{du} = \frac{1}{2}$

$\Rightarrow \frac{3}{2} \int e^{2t} = \frac{3}{2} \int e^{u} \cdot \frac{dt}{du} \cdot du$

$= \frac{3}{2} \left\{ \frac{1}{2} e^{2t} + c \right.$

$= \frac{3}{4} e^{2t} + c$

$\Rightarrow \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + c$

c) $\int x^2 \sin x \, dx$

$u = x^2$ ; $dv = \sin x$

$du = 2x \, dx$ ; $v = -\cos x$

$\int u dv = uv - \int v du$

$= x^2 \cdot -\cos x - \int -\cos x \cdot 2x dx$

$= -x^2 \cos x + 2x \sin x + c$

d) $\int \cos 5x \cos 6x \, dx$

Let $A = 5x$ and $B = 6x$

Recall that;

$\cos A \cos B = \frac{1}{2} \left( \cos(A+B) + \cos(A-B) \right)$

$= \frac{1}{2} \left( \cos(5x+6x) + \cos(5x-6x) \right)$

$$= \frac{1}{2}\left(\cos 11x - \cos x\right)$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\int\left(\cos 11x - \cos x\right) dx$$

$$\Rightarrow \frac{1}{2} \cdot \left(\frac{\sin 11x}{11} - \sin x\right)$$

$$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

$$\Rightarrow \int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

e) $\int \sin 7x \cos 2x \, dx$

Let $A = 7x$ and $B = 2x$

Recall that;

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$= \frac{1}{2}\left(\sin(7x+2x) + \sin(7x-2x)\right)$$

$$= \frac{1}{2}\left(\sin 9x + \sin 5x\right)$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int\left(\sin 9x + \sin 5x\right) dx$$

$$= \frac{1}{2}\int\left(\sin 9x + \sin 5x\right) dx$$

$$\Rightarrow \frac{1}{2} \cdot \left(\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right)$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$\Rightarrow \int \sin 7x \cos 2x \, dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$