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19/MHS01/108
MEDICINE AND SURGERY
MATH 104

Integrate the following functions

1) $2x^2 \ln x$

Let $u = \ln x$ $\quad du = \frac{1}{x}dx$

$dv = \ln x$ $\quad v = \frac{2}{3}x^3$

$\int u\,dv = uv - \int v\,du$

$\int 2x^2 \ln x = \ln x\left(\frac{2}{3}x^3\right) - \int \frac{2}{3}x^3 \cdot \frac{1}{x}dx$

$\int 2x^2 \ln x = \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^2 dx$

$\int 2x^2 \ln x = \frac{2}{3}x^3 \ln x - \frac{2}{3} \cdot \frac{x^3}{3} + c$

$\int 2x^2 \ln x = \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + c$

2) $3te^{2t}$

Let $u \cdot 3t$ $\quad dv = e^{2t}$

$du = 3dx$ $\quad v = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dx$

$\int 3te^{2t} = \frac{3}{2}te^{2t} - \frac{3}{2}\int e^{2t}dx$

$\int 3te^{2t} = \frac{3}{2}te^{2t} - \frac{3}{2} \cdot \frac{1}{2}e^{2t} + c$

$\int 3te^{2t} = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + c$

3) $\int x^2 \sin x$

$u = x^2 \qquad du = 2x\,dx$

$dv = \sin x \qquad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x = x^2(-\cos x) - \int -\cos x \cdot 2x\,dx$

$= -x^2 \cos x + \int \cos x\, 2x\,dx$

$= -x^2 \cos x + \quad u = 2x,\ du = 2\,dx$

$= -x^2 \cos x \qquad dv = \cos x \quad v = \sin x$

$= -x^2 \cos x \qquad u\,dv = uv - \int v\,du$

$u\,dv = 2x \sin x - \int \sin x\, 2\,dx$

$= -x^2 \cos x \qquad u\,dv = 2x \sin x - 2\int \sin x\,dx$

$u\,dv = 2x \sin x - 2(-\cos x) + c$

$\int x^2 \sin x = -x^2 \cos x + 2x \sin + 2\cos x + c$

4) $\cos 5x \cos 6x$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$A = 5x,\ B = 6x$

$\cos 5x \cos 6x = \frac{1}{2}\left[\cos(5x+6x) + \cos(5x-6x)\right]$

$= \frac{1}{2}(\cos 11x + \cos -x)$

$= \int \frac{1}{2}(\cos 11x + \cos x)\,dx$

$= \frac{1}{2}\int (\cos 11x + \cos x)\,dx$

$= \frac{1}{2}\int \cos 11x\,dx + \int \cos x\,dx$

$= \frac{1}{2}\left(\frac{1}{11}\sin x + \sin x\right) + c$

$= \frac{1}{22}\sin x + \frac{1}{2}\sin x + c$

$\therefore \int \cos 5x \cos 6x = \frac{1}{22}\sin x + \frac{1}{2}\sin x + c$

$\int \cos 5x \cos 6x = \frac{\sin x}{22} + \frac{\sin x}{2} + c$

5) $\sin 7x \cos 2x$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$A = 7x, \quad B = 2x$

$\sin 7x \cos 2x = \frac{1}{2}\left[\sin(7x+2x) + \sin(7x-2x)\right]$

$\sin 7x \cos 2x = \frac{1}{2}\int(\sin 9x + \sin 5x)\,dx$

$\sin 7x \cos 2x = \frac{1}{2}\int \sin 9x\,dx + \sin 5x\,dx$

$\sin 7x \cos 2x = \frac{1}{2}\left[-\frac{1}{9}\cos 9x + \left(\frac{-1}{5}\cos 5x\right)\right] + c$

$\int \sin 7x \cos 2x = \frac{-1}{18}\cos 9x - \frac{1}{10}\cos 5x + c$

$\therefore \int \sin 7x \cos 2x = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + c$