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MATRIC NO.:19/MHS01/309

COLLEGE: Medicine and Health Sciences

DEPARTMENT: Medicine and Surgery LEVEL: 100

Assignment Title: Calculus For MBBS Students ONLY

Course Title: General Mathematics III

Course Code: MAT 104

Question

Examine whether or not these pair of lines are perpendicular to each other. (1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ (2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ (3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

$$\textcircled{1} \quad y - 3x - 2 = 0 \quad \text{ie} \quad -3x + y - 2 = 0 \quad \text{---} \quad \textcircled{1} \quad a_1 = -3, b_1 = 1$$
$$3y + x + 9 = 0 \quad \text{ie} \quad x + 3y + 9 = 0 \quad \text{---} \quad \textcircled{2} \quad a_2 = 1, b_2 = 3$$

$$a_1 a_2 + b_1 b_2 = -3(1) + 1(3) = -3 + 3 = 0$$

$$a_1 a_2 + b_1 b_2 = 0$$

$$\therefore \textcircled{1} \perp \textcircled{2}$$

If product of slopes is -1 then lines are perpendicular.

$$m_1, m_2 = -1; \quad y = mx + c$$

$$\textcircled{1} \quad y - 3x - 2 = 0 \Rightarrow y = 3x + 2 \Rightarrow m_1 = 3$$

$$3y + x + 9 = 0 \Rightarrow y = -\frac{1}{3}x - 3 \Rightarrow m_2 = -\frac{1}{3}$$

$$m_1, m_2 = (3)\left(-\frac{1}{3}\right) = -1$$

\therefore The lines are perpendicular.

$$\textcircled{2} \quad 3y - 4 = 2x + 3 \Rightarrow y = \frac{2}{3}x + \frac{7}{3} \Rightarrow m_1 = \frac{2}{3}$$

$$y - 5 = x + 6 \Rightarrow y = x + 11 \Rightarrow m_2 = 1$$

$$m_1, m_2 = \left(\frac{2}{3}\right)(1) = \frac{2}{3} \neq -1$$

\therefore The lines are not perpendicular

$$\textcircled{3} \quad x^2 + y^2 + 3x + 4y - 11 = 0, (x_0, y_0) = (1, 2)$$

differentiate implicitly w.r.t x

$$2x + 2y \cdot y' + 3 \cdot 1 + 3 \cdot y' - 0 = 0$$

$$(2y + 3x) y' = -2x - 3y$$

$$y' = \frac{-2x - 3y}{2y + 3x}$$

Slope of tangent line $m_1 = y'(1, 2) = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$

Tangent line $y - y_0 = m_1(x - x_0)$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$y = \frac{-8}{7}x + \frac{22}{7}$$

Normal line and tangent line are perpendicular

$$\therefore m_1 \cdot m_2 = \frac{-8}{7} \cdot m_2 = -1$$

$\textcircled{1}$

$$\frac{-8}{7} \cdot m_2 = -1$$

$$m_2 = \frac{7}{8}$$

Normal line $y - y_0 = m_2(x - x_0)$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$y = \frac{7}{8}x + \frac{9}{8}$$