

① $2x^2 \ln x$

Solution:

$$u = \ln x \quad dv = 2x^2$$

$$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{2x^3}{3}$$

$$\int u dv = uv - \int v du$$

$$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$$

$$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$$

$$= \frac{2x^3}{3} \ln x - \frac{2x^3}{3 \times 3} + C$$

$$\therefore \int 2x^2 \ln x dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$$

OR

$$\frac{2x^3}{3} \left(\ln x - \frac{1}{3} \right) + C_1$$

② $3te^{2t}$

Solution:

$$u = 3t \quad dv = e^{2t}$$

$$du = 3dx \quad v = \frac{1}{2} e^{2t}$$

Continuation of no. 2

$$\int u dv = uv - \int v du$$

$$\int 3te^{2t} = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 dt$$

$$\int 3te^{2t} = \frac{3t}{2} e^{2t} - \int \frac{3}{2} e^{2t} \cdot dt$$

$$= \frac{3t}{2} e^{2t} - \frac{1}{2} \cdot \frac{3}{2} e^{2t} + C$$

$$\therefore \int 3te^{2t} dt = \left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C$$

③ $x^2 \sin x$

Solution:

$$u = x^2 \quad dv = \sin x$$

$$\frac{du}{dx} = 2x \quad v = -\cos x$$

$$du = 2x dx$$

$$\int u dv = uv - \int v du$$

$$\int x^2 \sin x dx = +x^2 \cdot -\cos x - \int -\cos x \cdot 2x dx$$
$$= -x^2 \cos x + \int 2x \cos x dx$$

$$= -x^2 \cos x + \left[\begin{array}{l} u = 2x \quad dv = \cos x \\ du = 2dx \quad v = \sin x \end{array} \right]$$

$$= -x^2 \cos x + uv - \int v du$$

$$= -x^2 \cos x + 2x \sin x - \int \sin x \cdot 2 dx$$

$$\int x^2 \sin x = -x^2 \cos x + 2x \sin x - \int 2 \sin x$$

$$\therefore \int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x$$

4. $\cos 5x \cos 6x$

Solution:

$$A = 5x, B = 6x$$

Recall that;

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) - \cos(A-B)]$$

$$= \frac{1}{2} [\cos 11x + \cos x]$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \left[\frac{\sin 11x}{11} + \sin x \right] + C$$

$$\therefore \int \cos 5x \cos 6x dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

5. $\sin 7x \cos 2x$

Solution:

$$A = 7x, B = 2x$$

Recall that;

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} \int \sin 9x + \sin 5x$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$\therefore \int \sin 7x \cos 2x dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$