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COURSE: GENERAL MATHEMATICS 104
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Integrate the following functions

1.) $2x^2 \ln x$

Solution

Let $u = \ln x$ $\qquad dv = 2x^2$

$du = \frac{1}{x} dx$

$$\int u\,dv = uv - \int v\,du$$

$$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$$

$$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{3 \times 3} + c$$

$$= 2x^2 \ln x\, dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c$$

OR

$$\frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$$

2) $\cancel{3te^{2t}}$ $\quad 3te^{2t}$

Solution

Let $u = 3t$ $\qquad dv = e^{2t}$

$du = 3dx$ $\qquad v = \frac{1}{2}e^{2t}$

$$\int u\,dv = uv - \int v\,du$$

$$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dx$$

$$\int 3te^{2t} = \frac{3t}{2}e^{2t} - \int \frac{3}{2}e^{2t} \cdot dx$$

$$= \frac{3t}{2}e^{2t} - \frac{1}{2} \cdot \frac{3}{2}e^{2t} + c$$

$$\therefore \int 3te^{2t} dt = \left[\frac{3t}{2}e^{2t} - \frac{3e^{2t}}{4}\right] + C$$

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3) $x^2 \sin x$

Solution

$u = x^2 \quad dv = \sin x$

$\frac{du}{x} = 2x \quad v = -\cos x$

$dx = 2x\,dx$

$\int u\,dv = uv - \int v\,dx$

$\int x^2 \sin x\,dx = +x^2 - \cos x - \int \cos x \cdot 2x\,dx$

$= -x^2 \cos x + \int 2x \cos x\,dx$

$= -x^2 \cos x + \boxed{\begin{matrix} u = 2x & dv = \cos x \\ du = 2dx & v = \sin x \end{matrix}}$

$= -x^2 \cos x + uv - \int v\,du$

$= -x^2 \cos x + 2x \sin x - \int \sin x \cdot 2dx$

$\int x^2 \sin x = -x^2 \cos x + 2x \sin x - \int 2 \sin x$

$\therefore \int x^2 \sin x\,dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

4) $\cos 5x \cos 6x$

Solution

Let $y = 5x$, $z = 6x$

Recall that;

$\cos A \cos B = \frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$= \frac{1}{2}\int \cos 11x + \cos x$

$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\left[\frac{\sin 11x}{11} + \sin x\right] + C$

$\therefore \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{2} + \frac{\sin x}{2} + C$

5) $\sin 7x \cos 2x$

Solution

$y = 7x$, $z = 2x$

Recall that;

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\int \sin 9x + \sin 5x$

$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right] + C$

$\therefore \int \sin 7x \cos 2x\,dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$