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Dept: MBBS

1 Integrate the following functions.

a. $2x^2 \ln x$

$u = \ln x$, $dv = 2x^2$

$du = \frac{1}{x} dx$; $v = \frac{2x^3}{3}$

Recall that:

$\int u\,dv = uv - \int v\,du$

$\rightarrow = \left(\ln x \cdot \frac{2x^3}{3}\right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

$= \left(\ln x \cdot \frac{2}{3} x^3\right) - \int \frac{2x^3}{3} dx$

$= \left(\ln x \cdot \frac{2}{3} x^3\right) - \frac{2x^3}{9} + C$

$= \frac{2}{3} x^3 \left(\ln x - \frac{1}{3}\right) + C$

b. $3t\,e^{2t}$

$u = 3t$; $dv = e^{2t}$

$du = 3dx$; $v = e^{2t} = \frac{1}{2} e^{2t}$

$\int u\,dv = uv - \int v\,du$

$= 3t \frac{1e^{2t}}{2} - \int \frac{1e^{2t}}{2} \cdot 3$

$\rightarrow 3t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3$

e. $\int \sin 7x \cos 2x$

Let $A = 7x$ and $B = 2x$

Recall that:

$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

$= \frac{1}{2}\left(\sin(7x+2x) + \sin(7x-2x)\right)$

$= \frac{1}{2}\left(\sin 9x + \sin 5x\right)$

$\int \sin 7x \cos 2x\,dx = \frac{1}{2} \int \left(\sin 9x + \sin 5x\right) dx$

$= \frac{1}{2} \int \left(\sin 9x + \sin 5x\right) dx$

$= \frac{1}{2} \cdot \left(\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right)$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

$= \int \sin 7x \cos 2x\,dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$

$= \frac{3}{2}te^{2t} - \int \frac{e^{2t}}{2} 3$

$= \frac{3t}{2}e^{2t} - \int \frac{3}{2}e^{2t}$

$= \frac{3}{2}te^{2t} - \left[\frac{3}{2}\int e^{2t}\right]$

$= \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$

c. $\int x^2 \sin x$

$u = x^2$; $dv = \sin x$

$du = 2x\,dx$; $v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + 2x\sin x + C$

d. $\int \cos 5x \cos 6x$

Let $A = 5x$ and $B = 6x$

Recall that:

$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

$= \frac{1}{2}\left(\cos(5x+6x) + \cos(5x-6x)\right)$

$= \frac{1}{2}\left(\cos 11x - \cos x\right)$

$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int\left(\cos 11x - \cos x\right)dx$

$= \frac{1}{2}\cdot\left(\frac{\sin 11x}{11} - \sin x\right)$

$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$

$= \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$