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MATH 104

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19/MHS01/332

MBBS

Integrate the following functions

1) $\int 2x^2 \ln x \, dx$

let $u = \ln x$, $dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x}$, $v = \frac{2x^3}{3}$

$du = \frac{dx}{x}$

$\int 2x^2 \ln x \, dx = \ln x \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$

$= \frac{2x^3}{3}\left(\ln x - \frac{2}{3}\int x^2 dx\right)$

$= \frac{2}{3}x^3\left(\ln x - \frac{2}{3} \cdot \frac{x^3}{3}\right)$

$\int 2x^2 \ln x \, dx = \frac{2}{3}x^3\left(\ln x - \frac{2x^3}{9}\right) + C$

2) $\int 3te^{2t} dx$

sd

$3\int te^{2t} dx$

let $u = t$, $dv = e^{2t}$

$\frac{du}{dt} = 1$, $v = \frac{e^{2t}}{2}$

$du = dt$

$\int 3te^{2t} dx = t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot dt$

$\int 3te^{2t} dx = \frac{te^{2t}}{2} - \frac{1}{2}\int e^{2t}$

$= 3\left(\frac{te^{2t}}{2} - \frac{1}{2} \cdot \frac{e^{2t}}{2}\right)$

$= 3\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right) + C$

$\int 3te^{2t} dx = \left(\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}\right) + C$

~~3) $\int x^2 \sin x \, dx$~~

~~let $u = \sin x$, $dv = x^2$~~

~~$\frac{du}{dx} = \cos x$, $v = \frac{x^3}{3}$~~

~~$du = \cos x \, dx$~~

~~$\int x^2 \sin x \, dx = \sin x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cos x \, dx$~~

~~$= \frac{x^3}{3}(\sin x$~~

3) $\int x^2 \sin x$

let $u = x^2$ $dv = \sin x$

$\frac{du}{dx} = 2x$, $v = -\cos x$

$du = 2x \, dx$

$\int x^2 \sin x = x^2 \cdot -\cos x - \int -\cos x \, 2x \, dx$

$\int x^2 \sin x\,dx = -x^2\cos x - \int -2x\cos x\,dx$ — ①

Solving.

$\int -2x\cos x\,dx$

$-2\int x\cos x\,dx$

Let $u = x$, $dv = \cos x$

$\frac{du}{dx} = 1$, $v = \sin x$

$du = dx$

$-2\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= -2(x\sin x + \cos x)$

$\int -2x\cos x\,dx = -2x\sin x - 2\cos x$ — ②

$2x\sin x - x^2\cos x + 2\cos x + c$

Rewrite as

$= 2x\sin x + (2 - x^2)\cos x + c$

4) $\cos 5x \cos 6x$

Sol

$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\int \cos 5x\cos 6x = \frac{1}{2}\cos 11x + \cos -x$

$= \frac{1}{2}\int (\cos 11x - x\cos)\,dx$

$\frac{1}{2}\left[\frac{1}{11}\sin 11x - \frac{1}{x}\,x\sin\right]dx$

$\int \cos 5x\cos 6x = \frac{1}{22}\sin 11x + \frac{1}{2}\sin -x + c$

$\int \cos 5x\cos 6x = \frac{1}{22}\sin 11x - \frac{x}{2}\sin + c$

5) $\sin 7x\cos 2x$

Sol

$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\int \sin 7x\cos 2x = \frac{1}{2}\sin 9x + \sin 5x$

$= \frac{1}{2}\int (\sin 9x + \sin 5x)\,dx$

$= \frac{1}{2}\left[-\frac{1}{9}\cos 9x - \frac{1}{5}\cos 5x\right] + c$

$\int \sin 7x\cos 2x = -\frac{1}{18}\cos 9x - \frac{1}{5}\cos 5x + c$