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Integrate the following functions

$\int 2x^2 \ln x \, dx = \int u\,dv = uv - \int v\,du$

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{2x^3}{3}$

$du = \frac{1}{x}\,dx$

$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x}\,dx$

$\frac{2x^3}{3}\ln x - \frac{2}{3}\int x^2\,dx$

$\frac{2x^3}{3}\ln x - \frac{2}{3}\cdot\frac{x^3}{3}$

$\frac{2x^3}{3}\ln x - \frac{2x^3}{9}$

$\int 2x^2 \ln x\,dx = \frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$

2 $\int 3te^{2t}\,dt$

$u = 3t \qquad dv = e^{2t}$

$\frac{du}{dt} = 3 \qquad v = \frac{1}{2}e^{2t}$

$du = 3dt$

$\int u\,dv = uv - \int v\,du$

$= 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dt$

$= \frac{3t}{2}e^{2t} - \frac{3}{2}\int e^{2t}\,dt$

$= \frac{3t}{2}e^{2t} - \frac{3}{2}\cdot\frac{1}{2}e^{2t} + C$

$\int 3te^{2t}\,dt = \frac{3t}{2}e^{2t} - \frac{3}{4}e^{2t} + C$

3) $\int x^2 \sin x\,dx$

$u = x^2$ $\quad dv = \sin x$

$\frac{du}{dx} = 2x$ $\quad v = -\cos x$

$du = 2x\,dx$

$\int u\,dv = uv - \int v\,du$

$= x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + 2\int x\cos x\,dx$

$= -x^2\cos x + x\sin x - \int \sin x\,dx$

$u = x$ $\quad dv = \cos x$

$\frac{du}{dx} = 1$ $\quad v = \sin x$

$du = dx$

$= -x^2\cos x + x\sin x - \int \sin x\,dx$

$= -x^2\cos x + x\sin x + \cos x + C$

$\therefore \int x^2 \sin x\,dx = -x\cos x + x^2\sin x + \cos x + C$

4) $\int \cos 5x \cos 6x\,dx$

$A = 5x, \quad B = 6x \quad A+B = 5x+6x = 11x, \quad A-B = -x$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}[\cos 11x - \cos x]$

$= \frac{1}{2}\left[\frac{\sin 11x}{11} - \sin x\right]$

$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$

5) $\int \sin A \cos B\,x\,dx$

$A = 7x, \quad B = 2x$

$A+B = 7x+2x = 9x, \quad A-B = 7x-2x = 5x$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$