ITYOACHIMIN FAITH MNENA
19/MHS01/210
MBBS
MAT 104
Integrate the following functions.

1. $2x^2 \ln x$

Let $u = \ln x$ $\quad du = \frac{1}{x} dx$

$dv = \ln x$ $\quad v = \frac{2}{3} x^3$

$$\int u\,dv = uv - \int v\,du$$
$$\int 2x^2 \ln x = \ln x \left(\frac{2}{3}x^3\right) - \int \frac{2}{3} x^3 \cdot \frac{1}{x} dx$$
$$\int 2x^2 \ln x = \frac{2}{3}x^3 \ln x - \frac{2}{3} \int x^2 dx$$
$$\int 2x^2 \ln x = \frac{2}{3} x^3 \ln x - \frac{2}{3} \cdot \frac{x^3}{3} + c$$
$$\int 2x^2 \ln x = \frac{2}{3} x^3 \ln x - \frac{2}{9} x^3 + c.$$

2. $3te^{2t}$

Let $u = 3t$ $\quad dv = e^{2t}$

$du = 3dx$ $\quad v \ \frac{1}{2} e^{2t}$

$$\int u\,dv = uv - \int v\,du$$
$$\int 3te^{2t} = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3dx$$
$$\int 3te^{2t} = \frac{3}{2} te^{2t} - \frac{3}{2} \int e^{2t} dx$$
$$\int 3te^{2t} = \frac{3}{2} te^{2t} - \frac{3}{2} \cdot \frac{1}{2} e^{2t} + c$$
$$\int 3te^{2t} = \frac{3}{2} te^{2t} - \frac{3}{4} e^{2t} + c.$$

3. $x^2 \sin x$

$u = x^2$, $\quad du = 2x\,dx$

$d\cancel{x} = \sin x$ $\quad v = -\cos x$

$$\int u\,dv = uv - \int v\,du.$$
$$\int x^2 \sin x = x^2(-\cos x) - \int \cos x \cdot 2x\,dx$$
$$= -x^2 \cos x + \int \cos x\, 2x\, dx$$
$$= -x^2 \cos x + \left\{ \begin{array}{l} u = 2x,\ du = 2dx \\ dv = \cos x,\ v = \sin x \\ u\,dv = uv - \int v\,du \\ 2x \sin x - \int \sin x\, 2dx \\ 2x \sin x - 2\int \sin x\, dx \\ 2x \sin x - 2(-\cos x) \end{array} \right\}$$
$$= -x^2 \cos x$$
$$= -x^2 \cos x$$
$$= -x^2 \cos x +$$
$$-x^2 \cos x +$$
$$-x^2 \cos x +$$
$$\int x^2 \sin x = -x^2 \cos x + 2x \sin x + 2\cos x + c$$

(4)

$\cos 5x \cos 6x$

$\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$

$A = 5x, \quad B = 6x$

$\cos 5x \cos 6x = \frac{1}{2}(\cos(5x+6x) + \cos(5x-6x))$

$= \frac{1}{2}(\cos 11x + \cos(-x))$

$\int(\frac{1}{2}\cos 11x + \cos x)\,dx$

$= \frac{1}{2}\int(\cos 11x + \cos x)\,dx$

$= \frac{1}{2}\int \cos 11x\,dx + \int \cos x\,dx$

$= \frac{1}{2}(\frac{1}{11}\sin 11x + \sin x) + C$

$= \frac{1}{22}\sin 11x + \frac{1}{2}\sin x + C$

$\therefore \int \cos 5x \cos 6x = \frac{1}{22}\sin 11x + \frac{1}{2}\sin x + C$

$\int \cos 5x \cos 6x = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$

(5)

$\sin 7x \cos 2x$

$\sin A \cos B = \frac{1}{2}(\sin(A+B) + (\sin(A-B))$

$A = 7x, \quad B = 2x$

$\sin 7x \cos 2x = \frac{1}{2}(\sin(7x+2x) + \sin(7x-2x))\}$

$\sin 7x \cos 2x = \frac{1}{2}\int(\sin 9x + \sin 5x)\,dx$

$\sin 7x \cos 2x = \frac{1}{2}\int \sin 9x\,dx + \int \sin 5x\,dx$

$\sin 7x \cos 2x = \frac{1}{2}(-\frac{1}{9}\cos 9x + -\frac{1}{5}\cos 5x)$

$\sin 7x \cos 2x = -\frac{1}{18}\cos 9x - \frac{1}{10}\cos 5x + C$

$\therefore \int \sin 7x \cos 2x = \frac{-\cos 9x}{18} - \frac{\cos 5x}{-10} + C$