Kanu Kennedy Kenechukwu
MBBS
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1.) $\int 2x^2 \ln x$

let $u = \ln x$, $\frac{dv}{dx} = 2x^2$

$\frac{du}{dx} = \frac{1}{x}$, $V = \int 3x^2 = \frac{x^3}{3}$

$= \int 2x^2 \ln x \, dx = 2\int x^2 \ln x \, dx$

Using the formular $Udv = UV - Vdu$

$= \int 2x^2 \ln x = 2\left(\frac{x^3}{3}\ln x - \int \frac{x^3}{3} \cdot \frac{1}{x}\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\right) dx$

$= \left(\frac{x^3 \ln x}{3} - \frac{1}{3}\int \frac{x^3}{3}\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \cancel{\frac{1}{3}\int} \frac{x^3}{9}\right) dx$

$= \frac{2x^3 \ln x}{3} - \frac{2x^3}{9} dx$

$$= \frac{3x\,2x^3 \ln x - 2\int x^3\,dx}{9}$$

$$= \frac{2x^3(3\ln x - 1)}{9} + C$$

$$= \frac{2x^3(\ln x - 1)}{3} + C$$

$$\therefore \int 2x^2 \ln x = \frac{2x^3(\ln x - 1)}{3} + C$$

2.) $\int 3te^{2t}\,dt$

let $u = 3t$, $\frac{dv}{dt} = e^{2t}$

$\frac{du}{dt} = 3$

, $v = \int e^{2t} = \frac{1}{2}e^{2t}$

$$\int u\,dv = uv - \int v\,du$$

$$\int 3te^{2t}\,dt = \left[3t \cdot \tfrac{1}{2}e^{2t} - \int \tfrac{1}{2}e^{2t} \cdot 3\right) dx$$

$$\int 3te^{2t}\,dt = \left[\frac{3te^{2t}}{2} - \frac{1}{4}e^{2t} \cdot 3\right) dx$$

$$\int 3te^{2t}\,dt = \left[\frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}\right] dx$$

$$\int 3te^{2t}\,dt = \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t} + C$$

3.) $\int x^2 \sin x$

let $u = x^2$, $\frac{dv}{dx} = \sin x$

$\frac{du}{dx} = 2x$, $v = \int \sin x = -\cos x$

$UdV = Vv - Vdu$

$\int x^2 \sin x \, dx = [x^2 \cdot -\cos x - \int -\cos x \cdot 2x) \, dx$

$\int x^2 \sin x \, dx = [x^2 \cos x - \int -2x \cos x] \, dx$

using integration by parts; $u = x$, $\frac{dv}{dx} = \cos x$

$\frac{dv}{dx} = 1$, $v = \int \cos x = \sin x$

$\int x^2 \sin x \, dx = (-x^2 \cos x + 2 \int x \cos x) \, dx$

$\int x^2 \sin x \, dx = ((-x^2 \cos x + 2(x \sin x + \cos x)) \, dx$

$\int x^2 \sin x \, dx = ((-x^2 \cos x + 2x \sin x + 2\cos x)) \, dx$

$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x + C$

4.) $\int \cos 5x \cos 6x$

$A = 5x$, $B = 6x$

Recall; $\frac{1}{2} (\cos(A+B) + \cos(A-B))$

$= \frac{1}{2} (\cos(5x + 6x) + \cos(5x - 6x)) \, dx$

$= \frac{1}{2} (\cos 11x + \cos(-x)) \, dx$

$= \frac{1}{2} (\cos 11x - \cos x) \, dx$

$= \frac{1}{2} \left\{ \frac{-\sin 11x}{4} - \frac{(-\sin x)}{1} \right\} dx$

$$= \frac{1}{2}\left(\frac{-\sin 11x}{11} + \frac{\sin x}{1}\right)$$

$$= -\frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

5.) $\int \sin 7x \cos 2x$

$A = 7x$, $B = 2x$

Recall $= \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$$= \frac{1}{2}(\sin 7x + 2x) + \sin(7x - 2x))\,dx$$

$$= \frac{1}{2}(\sin 9x + \sin 5x)\,dx$$

$$= \frac{1}{2}(-\cos 9x + (-\cos 5x))$$

$$= \frac{1}{2}\left(\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right)$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10}$$