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1. Integration of functions

$2x^2 \ln x$

$\int 2x^2 \ln(x)\,dx$

$2\int x^2 \ln(x)\,dx$ - Apply linearity

$\int x^2 \ln(x)\,dx$

Integrate by parts $\int fg' = fg - \int f'g$

$f = \ln(x)$

$f' = 1/x$

$g' = x^2$

$g = x^3/3$

$= \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3}\,dx$

$\int x^2/3\,dx$

$= 1/3 \int x^2\,dx$

$\int x^2\,dx$

Apply power rule

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$ with $n = 2$:

$= x^3/3$

$1/3 \int x^2\,dx$

$= x^3/9$

$\frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3}\,dx$

$= \frac{x^3 \ln(x)}{3} - \frac{2x^3}{9}$

$\int 2x^2 \ln(x)\,dx$

$= \frac{2x^3 \ln(x)}{3} - \frac{2x^3}{9} + C$

$= \frac{2x^3(3\ln(x) - 1)}{9} + C$

$\cancel{3e^2}$ $3te^{2t}$

2. Apply linearity $= 3\int te^{2t}\,dt$

$\int te^{2t}\,dt$

Integrate by parts $\int fg' = fg - \int f'g$

$f = t,\ g' = e^{2t}$

$f' = 1 \quad g = e^{2t}/2$

$= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt$

$\int e^{2t}/2\,dt$

Substitute $u = 2t$

$du/dt = dt = \frac{1}{2}du$

$= \frac{1}{4}\int e^u du$

$\int e^u du$

Apply exponential rule

$\int a^u du = \frac{a^u}{\ln(a)}$ with $a = e$

$= e^u$

$\frac{1}{4}\int e^u du$

$= e^u/4$

Undo substitution $u = 2t$

$= \frac{e^{2t}}{4}$

$\frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt$

$= \frac{te^{2t}}{2} - \frac{e^{2t}}{4}$

$\int 3te^{2t}\,dt$

$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + c$

$= \frac{3(2t-1)e^{2t}}{4} + c$

3. $x^2 \sin x$

Integrate by parts: $\int fg' = fg - \int f'g$

$f = x^2, \; g' = \sin(x)$

$f' = 2x, \; g = -\cos(x)$

$= -x^2\cos(x) - \int -2x\cos(x)dx$

$\int -2x\cos(x)dx$

$= -2\int x\cos(x)dx$ Apply linearity

$\int x\cos(x)dx$

Integrate by parts: $\int fg' = fg - \int f'g$

$f = x \quad , \; g' = \cos(x)$

$f' = 1 \quad g = \sin(x)$

$= x\sin(x) - \int \sin(x)dx$

$\int \sin(x)dx$

$= -\cos(x)$

$x\sin(x) - \int \sin(x)dx$

$= x\sin(x) + \cos(x)$

$-2\int x\cos(x)dx$

$= -2x\sin(x) - 2\cos(x)$

$-x^2\cos(x) - \int -2x\cos(x)dx$

$= 2x\sin(x) - x^2\cos(x) + 2\cos(x)$

$\int x^2\sin(x)dx$

$= 2x\sin(x) - x^2\cos(x) + 2\cos(x) + C$

$= 2x\sin(x) + (2 - x^2)\cos(x) + C$

4.

4. $\cos 5x \cos 6x$

$$\int \cos(5x)\cos(6x)\,dx$$

Apply product-to-sum formulas:

$\sin(x)\sin(y) = 1/2(\cos(y-x) - \cos(y+x))$

$\sin^2(x) = 1/2(1-\cos(2x))$

$\cos(x)\cos(y) = 1/2(\cos(y+x)+\cos(y-x))$

$\cos^2(x) = 1/2(\cos(2x)+1)$

$\sin(x)\cos(y) = 1/2(\sin(y+x) - \sin(y-x))$

$\cos(x)\sin(x) = 1/2\sin(2x)$

$$= \int \frac{\cos(11x)+\cos(x)}{2}\,dx$$

Apply linearity

$$= 1/2\int\cos(11x)\,dx + 1/2\int\cos(x)\,dx$$

$$\int\cos(11x)\,dx$$

Substitue $u = 11x$

$du/dx = 11$

$dx = 1/11\, du$

$= 1/11\int\cos(u)\,du$

$\int\cos(u)\,du$

$= \sin(u)$

$1/11\int\cos(u)\,du$

Undo substitution $= \frac{\sin(u)}{11} = \frac{\sin(11x)}{11}$

$(u = 11x)$

$\int\cos(x)\,dx$

$= \sin(x)$

$1/2\int\cos(11x)\,dx + 1/2\int\cos(x)\,dx$

$$= \frac{\sin(11x)}{22} + \frac{\sin(x)}{2}$$

$$\int \frac{\sin(11x) + \sin(x)}{22} + C$$

5. $\sin 7x \cos 2x$

$\int \cos(2x)\sin(7x)\,dx$

Apply product-to-sum formulas:

$\sin(x)\sin(y) = 1/2(\cos(y-x) - \cos(y+x))$

$\sin^2(x) = 1/2(1-\cos(2x))$

$\cos(x)\cos(y) = 1/2(\cos(y+x) + \cos(y-x))$

$\cos^2(x) = 1/2(\cos(2x)+1)$

$\sin(x)\cos(y) = 1/2(\sin(y+x) - \sin(y-x))$

$\cos(x)\sin(x) = 1/2\sin(2x)$

$= \int \frac{\sin(9x) + \sin(5x)}{2}\,dx$

$= 1/2\int \sin(9x)\,dx + 1/2\int \sin(5x)\,dx$

$\int \sin(9x)\,dx$

$= \cancel{1/9\int}$ substitue $u = 9x$

$du/dx = 9$

$dx = 1/9\,du$

$= 1/9\int \sin(u)\,du$

$\int \sin(u)\,du$

$= -\cos(u)$

$= 1/9\int \sin(u)\,du$

$= \frac{-\cos(u)}{9}$

Undo subsititution $u = 9x$

$= \frac{-\cos(9x)}{9}$

$\int \sin(5x)\,dx$

Subbstitue $u = 5x$

$du/dx = 5$

$dx = 1/5\,du$

$= 1/5\int \sin(u)\,du$

$= -\frac{\cos(u)}{5}$

Undo substitution $u = 5x$

$$= \frac{-\cos(5x)}{5}$$

$$\frac{1}{2}\int \sin(9x)\,dx + \frac{1}{2}\int \sin(5x)\,dx$$

$$= -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10}$$

$$\int \cos(2x)\sin(7x)\,dx$$

$$= -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$