

$$\int u dv = uv - \int v du$$

$$u = \ln x \quad dv = \frac{1}{3} dx$$

$$du = \frac{1}{x} dx$$

$$v = \frac{2x^3}{3}$$

$$\int \ln x \cdot \frac{2}{3} x^3 - \int \frac{2}{3} x^3 \cdot \frac{1}{x} dx$$

$$\int \ln x \cdot \frac{2}{3} x^3 - \int \frac{2}{3} x^2 dx$$

$$\int \ln x \cdot \frac{2}{3} x^3 - \frac{2}{3} \int \left(\frac{x^3}{3} \right) + C$$

$$\int 2x^2 \ln x = \frac{2}{3} x^3 (\ln x - \frac{1}{3}) + C$$

$$\int 3t e^{2t}$$

$$u = 3t \quad du = 3 dt$$

$$dv = e^{2t} \quad v = \frac{1}{2} e^{2t}$$

$$\int u dv = uv - \int v du$$

$$3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 dt$$

$$\frac{3t}{2} e^{2t} - \int \frac{3}{2} e^{2t}$$

$$\frac{3t}{2} e^{2t} - \left(\frac{1}{2} \times \frac{3}{2} e^{2t} \right)$$

$$\frac{3t}{2} e^{2t} - \frac{3}{4} e^{2t}$$

$$\int x^2 \sin x \, dx$$

$$u = x^2 \quad du = 2x \, dx$$

$$dv = \sin x \quad v = -\cos x$$

$$uv = \int v \, du$$

$$x^2 \cos x - \int -\cos x \cdot 2x \, dx$$

$$u = 2x \quad du = 2$$

$$dv = -\cos x \quad v = \sin x$$

$$-x^2 \cos x - [uv - \int v \, du]$$

$$-x^2 \cos x - [2x \sin x - \int 2 \sin x]$$

$$-x^2 \cos x - 2x \sin x - 2(-\cos x) + C$$

$$-x^2 \cos x - 2x \sin x + 2 \cos x + C$$

$$\cos 5x \cos 6x$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\cos 5x \cos 6x = \frac{1}{2} [\cos(5x+6x) + \cos(5x-6x)]$$

$$= \frac{1}{2} [\cos(11x) + \cos(-x)]$$

$$\int \cos 5x \cos 6x \, dx = \int \frac{1}{2} [\cos(11x) + \cos(-x)] \, dx$$

$$= \frac{1}{2} \int \cos(11x) + \cos(-x) \, dx$$

$$= \frac{1}{2} \left[\frac{-\sin(11x)}{11} - \frac{1}{1} + \frac{\sin(-x)}{-1} - \frac{1}{-1} \right]$$

$$= \left[\frac{-\sin 11x}{22} + \frac{\sin x}{2} \right] + C$$

$$\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

$$\int \sin 7x \cos 2x$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\sin 7x \cos 2x = \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$= \frac{1}{2} [\sin(9x) + \sin(5x)]$$

$$\int \sin 7x \cos 2x = \int \frac{1}{2} \sin(9x) + \sin(5x) dx$$

$$= \frac{1}{2} \int \sin(9x) + \sin(5x) dx$$

$$= \frac{1}{2} \left[-\cos(9x) \times \frac{1}{9} + -\cos(5x) \times \frac{1}{5} \right]$$

$$= \frac{1}{2} \left[-\frac{\cos(9x)}{9} + -\frac{\cos(5x)}{5} \right]$$

$$= -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$