MAT 104 $3te^{2t}$

$x^2\sin x$

MBBS

UNUANE-HENRY

OSEREMHEN C.

$\cos 5x\ 6x$

$\sin 7x\ \cos$ 19/MHS01/424

Integrate the following

a $2x^2\ln x$

Soln

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

$dv = 2x^2$

$\int dv = \int 2x^2$

$v = \frac{2x^{2+1}}{2+1} = \frac{2x^3}{3}$

$$\therefore \int 2x^2\ln x = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^{\cancel{3}\,2}}{3} \times \frac{1}{\cancel{x}}\,dx$$

$$= \frac{2x^3}{3}\ln x - \int \frac{2x^2}{3}\,dx$$

$$= \frac{2x^3\ln x}{3} - \frac{2}{3}\int x^2\,dx$$

$$= \tfrac{2}{3}x^3\ln x - \tfrac{2}{3}\left[\frac{x^3}{3}\right] + C$$

$$= \tfrac{2}{3}x^3\ln x - 2x^3/9 + C$$

$$= \cancel{2/3((x^3}$$

$$\int 2x^2\ln x = \frac{2x^3}{3}\left(\ln x - \tfrac{1}{3}\right) + C$$

b $3te^{2t}$

Soln

Let $u = 3t$

$\frac{du}{dt} = 3$

$du = 3dt$

$dv = e^{2t}$

$\int dv = \int e^{2t}$

$v = \frac{e^{2t}}{2}$

$\cancel{\int 3te^{2t} = ut - \int t\,dv}$

$$\int 3te^{2t} = uv - \int v\,du$$

$$\int 3te^{2t} = 3t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3\,dt$$

$$= \frac{3te^{2t}}{2} - \int \frac{3}{2} e^{2t}\, dt$$

$$= \frac{3te^{2t}}{2} - \frac{3}{2}\int e^{2t}\, dt$$

$$= \frac{3te^{2t}}{2} - \frac{3}{2}\left[\frac{1}{2} \cdot e^{2t}\right] + C$$

$$= \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t} + C$$

$$= \frac{3e^{2t}}{2}\left[t - \frac{1}{2}\right] + c$$

c $x^2 \sin x$

Soln

let $u = x^2$ $\qquad$ $dv = \sin x$

$\frac{du}{dx} = 2x$ $\qquad$ $\int dv = \int \sin x$

$v = -\cos x$

$du = 2x\,dx$

$$\therefore \int x^2 \sin x = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x\, dx$$

$$= -x^2\cos x + \int 2x \cos x\, dx$$

$$= -x^2 \cos x + 2\int x \cos x\, dx$$

$$= -x^2 \cos x + 2\left[\begin{array}{ll} u = x & dv = \cos x \\ \frac{du}{dx} = 1 & \int dv = \cos x \\ du = dx & v = \sin x \end{array}\right]$$

$$= -x^2 \cos x + 2\left[x \sin x - \int \sin x\, dx\right]$$

$$= -x^2 \cos x + 2\left[x \sin x - (-\cos x)\right] + C$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + C$$

d $\cos 5x \cos 6x$

$A = 5x,\ B = 6x$

$$\int \cos A \cos B = \int \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$\int \cos 5x \cos 6x = \int \frac{1}{2} \cos(11x) + \cos(-x)$$

$$\int \cos 5x \cos 6x = \frac{1}{2}\int \cos 11x + \cos(-x)$$

$$= \frac{1}{2}\left[\frac{\sin 11x}{11} + \left(- \frac{\sin(-x)}{1} \right) \right] + c$$

$$= \frac{1}{2}\left[\frac{\sin 11x}{11} - \sin(-x) \right] + c$$

$$= \frac{\sin 11x}{22} - \frac{\sin(-x)}{2} + c$$

5 $\sin 7x \cos 2x$

Let $A = 7x$, $B = 2x$

$$\int \sin A \cos B = \int \frac{1}{2} \sin(A+B) + \sin(A-B)$$

$$\int \sin 7x \cos 2x = \frac{1}{2}\int \sin 9x + \sin 5x$$

$$= \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + c$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + c$$