MAT 104 (ASSIGNMENT

Name: Tobby, Glory Inyang.
Department: Medicine and Surgery (MBBS)
Matric Number: 19/MHS01/406
Serial Number: 008

Integrate the following functions:

1) $2x^2 \ln x$

$\int 2x^2 \ln(x)\,dx$

$2\int x^2 \ln(x)\,dx$

Apply integration by parts: $u = \ln(x)$, $v' = x^2$

$2\left(\frac{1}{3}x^3 \ln(x) - \int \frac{x^2}{3}dx\right)$, $\int \frac{x^2}{3}dx = \frac{x^3}{9}$

$2\left(\frac{1}{3}x^3 \ln(x) - \frac{x^3}{9}\right)$

$$\boxed{\therefore \int 2x^2 \ln(x)\,dx \Rightarrow 2\left(\frac{1}{3}x^3 \ln(x) - \frac{x^3}{9}\right) + C}$$

2) $3te^{2t}$

$\int 3te^{2t} \Rightarrow 3\int te^{2t}\,dt$

let $u = 2t$

$3\int \frac{e^u u}{4}\,du \Rightarrow 3 \cdot \frac{1}{4}\int e^u u\,du$

$u = u$, $v' = e^u$

$3 \cdot \frac{1}{4}\left(e^u u - \int e^u du\right)$

$\int e^u du = e^u$

$3 \cdot \frac{1}{4}(e^u u - e^u)$

Substitute back $u = 2t$.

$3 \cdot \frac{1}{4}(e^{2t} \cdot 2t - e^{2t})$

Simplify $3 \cdot \frac{1}{4}\left(e^{2t} \cdot 2t - e^{2t}\right) \Rightarrow \frac{3}{4}\left(2e^{2t}t - e^{2t}\right)$

$$\boxed{\therefore \int 3te^{2t} \Rightarrow \frac{3}{4}\left(2e^{2t}t - e^{2t}\right) + C}$$

3) $x^2 \sin x$

$\int x^2 \sin(x)\,dx$

Apply integration by parts

$u = x^2,\ v' = \sin(x)$

$\Rightarrow -x^2\cos(x) - \int -2x\cos(x)\,dx$

$\int -2\cos(x)\,dx = -2(x\sin(x) + \cos(x))$

$\Rightarrow -x^2\cos(x) - (-2(x\sin(x) + \cos(x))$

Simplify $-x^2\cos(x) + 2(x\sin(x) + \cos(x))$

$\boxed{\therefore \int x^2\sin(x)\,dx \Rightarrow -x^2\cos(x) + 2(x\sin(x) + \cos(x)) + C}$

4) $\cos 5x \cos 6x$

$\int \cos 5x \cos 6x\,dx$

$\int \frac{\cos(5x+6x) + \cos(5x-6x)}{2}\,dx$

$\frac{1}{2}\int \cos(5x+6x) + \cos(5x-6x)\,dx \Rightarrow \frac{1}{2}\int \cos(11x) + \cos(x)\,dx$

$\frac{1}{2}\left(\int \cos(11x)\,dx + \int \cos(x)\,dx\right)$

$\int \cos 11x\,dx = \frac{1}{11}\sin(11x)$ and $\int \cos(x)\,dx = \sin(x)$

$\Rightarrow \frac{1}{2}\left(\frac{1}{11}\sin(11x) + \sin(x)\right)$

$\boxed{\therefore \int \cos 5x \cos 6x\,dx \Rightarrow \frac{1}{2}\left(\frac{1}{11}\sin(11x) + \sin(x)\right) + C}$

5) $\sin 7x \cos 2x$

$\int \sin 7x \cos 2x$

$\int \sin 7x \cos 2x \Rightarrow \int \frac{\sin(7x+2x) + \sin(7x-2x)}{2}\,dx$

$\Rightarrow \frac{1}{2}\int \sin(7x+2x) + \sin(7x-2x)\,dx \Rightarrow \frac{1}{2}\left(\int \sin(7x+2x)\,dx + \int \sin(7x-2x)\,dx\right)$

$\Rightarrow \frac{1}{2}\left(\int \sin(9x)\,dx + \int \sin(5x)\,dx\right)$

$\int \sin(9x)\,dx = -\frac{1}{9}\cos(9x)$ and $\int \sin(5x)\,dx = -\frac{1}{5}\cos(5x)$

$$\Rightarrow \frac{1}{2}\left(-\frac{1}{9}\cos(9x) - \frac{1}{5}\cos(5x)\right)$$

$$\boxed{\therefore \int \sin 7x \cos 2x \, dx \Rightarrow \frac{1}{2}\left(-\frac{1}{9}\cos(9x) - \frac{1}{5}\cos(5x)\right) + C}$$