

$$1) \int 2x^2 \ln x \, dx$$

$$f \, du = UV - \int V \, du$$

$$U = 2x^2, \, du = 4x \, dx$$

$$dV = \ln x \, dx \quad V = x \ln x - x$$

$$\int 2x^2 \ln x \, dx = 2x^2 \times (x \ln x - x) - \int x \ln x - x \times 4x \, dx$$

$$\int 2x^2 \ln x \, dx = 2x^3 \ln x - 2x^3 - \int 4x^2 \ln x - 4x^2$$

$$\int 2x^2 \ln x \, dx = 2x^3 \ln x - 2x^3 - \int 4x^2 (\ln x - 1)$$

$$\int 2x^2 \ln x \, dx = 2x^3 \ln x - 2x^3 - \frac{4x^3}{3} (\ln x - 1)$$

$$\int 2x^2 \ln x \, dx = 2x^3 \ln x - 2x^3 - \frac{4x^3}{3} (\ln x - 1) - \frac{4x^3}{3} (\ln x - 1) - 2x^3$$

$$\int 2x^2 \ln x \, dx = 2x^3 \ln x - 2x^3 - \frac{4x^4 \ln x}{3} - \frac{8x^4}{3}$$

$$\int 2x^2 \ln x \, dx = 2x^3 \left(\ln x - \frac{4x \ln x}{3} - \frac{4x}{3} - 1 \right) + C$$

$$2.) \int 3te^{2t} dt$$

$$\int u dv = uv - \int v du$$

$$u = 3t, \quad du = 3 \quad dv = e^{2t} dt \quad v = \frac{e^{2t}}{2}$$

$$\int 3te^{2t} dt = 3t \times \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \times 3$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \int \frac{3e^{2t}}{2}$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \frac{3}{2} \int e^{2t}$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C \quad \text{(or } \frac{3e^{2t}}{2} \left(t - \frac{1}{2} \right) + C)$$

$$3) \int x^2 \sin x dx$$

$$u = x^2 \quad du = 2x \quad dv = \sin x dx \quad v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$\int x^2 \sin x dx = x^2 \times -\cos x - \int -\cos x \times 2x$$

$$\int x^2 \sin x dx = x^2 \times -\cos x + \int \cos 2x \cos x$$

$$\int 2x \cos x = 2x \times \sin x - \int \sin x \times 2 \quad (u=2x, dv=\cos x, v=\sin x)$$

$$\int 2x \cos x = 2x \sin x - 2 \int \sin x \quad du=2$$

$$\int 2x \cos x = 2x \sin x + 2 \cos x$$

$$\therefore \int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$\begin{aligned} 4) \int \cos 5x \cos 6x dx &= \frac{1}{2} \int [\cos(A+B) + \cos(A-B)] \quad (A=5x, B=6x) \\ &= \frac{1}{2} \int [\cos(5x+6x) + \cos(5x-6x)] \\ &= \frac{1}{2} \int [\cos 11x + \cos x] \\ &= \frac{1}{2} \left[\frac{\sin 11x}{11} - \sin x \right] \\ &= \frac{\sin 11x}{22} - \frac{\sin x}{2} \end{aligned}$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)] \quad (A=7x, B=2x)$$

$$= \frac{1}{2} (\sin 9x + \sin 5x)$$

$$= \frac{1}{2} \left[-\frac{\cos 9x}{9} + \frac{\cos 5x}{5} \right]$$

$$= -\frac{\cos 9x}{18} + \frac{\cos 5x}{10}$$