

MATHS ASSIGNMENT

1) $2x^2 \ln x$

$u = \ln x \quad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{2x^3}{3}$

$du = \frac{1}{x} dx$

$\int u dv = uv - \int v du$

$= \ln x \times \frac{2x^3}{3} - \int \frac{2x^3}{3} \times \frac{1}{x} dx$

$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{3} \times \frac{1}{3} + C$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$

$\int 2x^2 \ln x = \frac{2x^3}{3} \left[\ln x - \frac{1}{3} \right] + C$

$$2. \int 3t e^{2t} dt \rightarrow I$$

Solution

$$u = 3t \quad dv = e^{2t}$$

$$\frac{du}{dt} = 3 \quad v = \frac{1}{2} e^{2t}$$

$$du = 3 dt$$

$$\int u dv = uv - \int v du$$

$$= 3t \times \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \times 3 dt$$

$$= \frac{3}{2} t e^{2t} - \int \frac{3 e^{2t}}{2} dt$$

$$= \frac{3}{2} te^{2t} - \frac{3}{2} \int e^{2t} dt$$

$$= \frac{3}{2} te^{2t} - \frac{3}{2} \times \frac{1}{2} e^{2t} + C$$

$$\int 3te^{2t} = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

3. $\int x^2 \sin x dx$

$$u = x^2 \quad dv = \sin x$$

$$du = 2x dx \quad v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$= x^2 \cdot -\cos x - \int -\cos x \cdot 2x dx$$

$$= -\cos x (x^2) + \int 2x \cos x dx \quad \dots \textcircled{1}$$

for

$$\therefore \int 2x \cos x dx \left[\begin{array}{l} u = 2x \quad dv = \cos x \\ du = 2 dx \quad v = \sin x \end{array} \right]$$

$$\rightarrow = 2x \sin x + 2x \sin x = 2x \sin x - \int \sin x \cdot 2 dx$$

Put it in equation 1

$$= -\cos x = -\cos x (x^2) + 2x \sin x - \int \sin x \cdot 2 dx$$

$$= -x^2 \cos x + 2x \sin x - 2x \cos x + C$$

$$\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4. \int \cos 5x \cos 6x \, dx$$

Solution

$$A = 5x \quad B = 6x$$

Recall that:-

$$\begin{aligned} \cos A \cos B &= \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ &= \frac{1}{2} [\cos 11x + \cos x] \end{aligned}$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2} \int [\sin 11x + \sin x]$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2} \int \cos 11x - \cos x \, dx$$

$$= \frac{1}{2} \left[\frac{\sin 11x}{11} - \sin x \right] + C$$

$$\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

$$5. \int \sin 7x \cos 2x \, dx$$

Solution

$$A = 7x, \quad b = 2x$$

$$\sin 7x \cos 2x = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\begin{aligned} \int \sin 7x \cos 2x \, dx &= \frac{1}{2} \int \sin 9x + \sin 5x \, dx \\ &= \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C \end{aligned}$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{5} - \frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$