$2x^2 \ln x$

$\int u\,dv = uv - \int v\,du$

LIATE

$u = \ln x \qquad dv = 2x^2$

$du = \frac{1}{x}dx \qquad v = \frac{2x^3}{3}$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x}\,dx$

$= \frac{2x^3}{3}\ln x - \int \frac{2x^3}{3x}\,dx$

$\frac{2x^3}{3}\ln x - \frac{2}{3}\int x^2\,dx = \frac{2x^3}{3}\ln x - \frac{2x^3}{9} + C$

2. $3t\,e^{2t}$

$u = 3t, \quad du = 3\,dt$

$v = e^{2t}, \quad dv = e^{2t}$

$3t \cdot e^{2t} - \int e^{2t} \cdot 3\,dt = 3t \cdot e^{2t} - 3\int e^{2t}\,dt$

$= 3t\,e^{2t} - 3e^{2t} + C$

4. $\cos 5x \cos 6x$

$$\cos A \cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)]$$

$$\cos 5 \cos 6x = \frac{1}{2}\{\cos(5x+6x) + \cos(5x-6x]$$

$$= \frac{1}{2}[\cos 11x + \cos x]$$

$$\int \cos 5x \cos 6x = \int\{\frac{1}{2}\{\cos 11x - \cos x]]$$

$$= \frac{1}{2}\left[\frac{\sin 11x}{11} + \frac{\sin x}{x}\right]$$

$$= -\frac{1}{2}\left[\frac{\sin 11x}{11} + - \frac{\sin x}{x}\right] + C$$

$$= -\frac{\sin 11x}{22} - \frac{\sin x}{2x} + C$$

5 $\sin 7x \cos 2x$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin 7x \cos 2x = \frac{1}{2}\left[\sin(9x) + \sin(5x)\right]$$

$$\int \sin 7x \cos 2x = \int \left[\frac{1}{2}\left[\sin 9x + \sin 5x\right]\right]$$

$$\int \sin 7x \cos 2x = -\frac{1}{2}\left[\frac{\cos 9x}{9} + \frac{\cos 5x}{5}\right] + c$$

$$-\left[\frac{\cos 9x}{18} + \frac{\cos 5x}{10}\right] + c$$

3. $x^2 \sin x$

$u = x^2, \ du = x^3/3, \ dv = \sin x, \ dv = -\cos x$

$$\int x^2 \sin x = x^2 \cdot -\cos x - \int -\cos x \cdot \frac{x^3}{3}$$

$$\int x^2 \sin x = -(x^2)(\cos x) - \frac{1}{3} \sin x \int x^3$$

$$\int x^2 \sin x = -(x^2)(\cos x) - \frac{1}{12}(\sin x)(x^4)$$