Image

1. $\int 2x^2 \ln x \, dx$

Solution

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$, $du = \frac{1}{x} dx$

$dv = 2x^2 dx$

$\int dv = \int 2x^2 dx$

$v = \frac{2x^3}{3}$

$\int u \, dv = uv - \int v \, du$

$\int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \int x^2 dx$

$= \frac{2}{3} x^3 \ln x - \frac{2}{3} \cdot \frac{x^3}{3} + c$

$= \frac{2}{3} x^3 \ln x - \frac{2x^3}{9} + c$

$\therefore \int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \left(\ln x - \frac{1}{3} \right) + c$

2. $\int 3t \, e^{2t} \, dt$

Solution.

$u = 3t$

$\frac{du}{dt} = 3$, $du = 3dt$

$dv = e^{2t} dt$

$v = \frac{e^{2t}}{2}$

$\int u \, dv = uv - \int v \, du$

$\int 3t \, e^{2t} dt = \frac{3t}{2} (e^{2t}) - \frac{3}{2} \int e^{2t} dt$

$= \frac{3t}{2} (e^{2t}) - \frac{3}{2} \frac{e^{2t}}{2} + c$

$\int 3t \, e^{2t} dt = \frac{3t}{2} (e^{2t}) - \frac{3}{4} e^{2t} + c$

$\therefore \int 3t \, e^{2t} dt = \frac{3}{2} e^{2t} \left(t - \frac{1}{2} \right) + c$

3. $\int x^2 \sin x \, dx$

~~u~~ Solution

$u = x^2$

$du = 2x \, dx$

$dv = \sin x \, dx$

$v = -\cos x$

$\int u \, dv = uv - \int v \, du$

$\int x^2 \sin x \, dx = -x^2 \cos x - \int (-\cos x)(2x \, dx)$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx$

but $\int x \cos x \, dx = ?$

$\int x\cos x\,dx$

$u = x$

$du = dx$

$dv = \cos x\,dx$

$v = \sin x$

$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x\sin x - (-\cos x)$

$= x\sin x + \cos x$

$\therefore \int x^2 \sin x\,dx = -x^2\cos x + 2(x\sin x + \cos x) + c$

$\boxed{\int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + c}$

4 $\int \cos 5x \cos 6x\,dx$

Solution

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$A = 5x,\ B = 6x$

$\cos 5x \cos 6x = \frac{1}{2}\left[\cos(5x+6x) + \cos(5x-6x)\right]$

$= \frac{1}{2}\left[\cos 11x + \cos(-x)\right]$

$\cos(-x) = \cos x$

$= \frac{1}{2}\left[\cos 11x + \cos x\right]$

$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int (\cos 11x + \cos x)\,dx$

$= \frac{1}{2}\left(\frac{\sin 11x}{11} + \frac{\sin x}{1}\right) + c$

$\therefore \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + c$

5 $\int \sin 7x \cos 2x\,dx$

Solution

$\sin 7x \cos 2x = \frac{1}{2}\left[\sin 9x + \sin 5x\right]$

$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\int\left[\sin 9x + \sin 5x\right]dx$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right] + c$

$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + c$

$\therefore \int \sin 7x \cos 2x\,dx =$

$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + c$