

a $2x^2 \ln x$

Let $u = \ln x$ Soln

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

$$dv = 2x^2$$

$$\int dv = \int 2x^2$$

$$v = \frac{2x^{2+1}}{2+1} = \frac{2x^3}{3} //$$

$$\therefore \int 2x^2 \ln x = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \times \frac{1}{x} dx$$

$$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$$

$$= \frac{2x^3}{3} \ln x - \frac{2}{3} \int x^2 dx$$

$$= \frac{2}{3} x^3 \ln x - \frac{2}{3} \left[\frac{x^3}{3} \right] + C$$

$$= \frac{2}{3} x^3 \ln x - \frac{2x^3}{9} + C$$

$$= \text{A/B/V/P}$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} (\ln x - \frac{1}{3}) + C$$

b $3te^{2t}$

Let $u = 3t$ Soln

$$\frac{du}{dt} = 3$$

$$du = 3dt$$

$$dv = e^{2t}$$

$$\int dv = \int e^{2t}$$

$$v = \frac{e^{2t}}{2} //$$

$$\int 3te^{2t} = ut - \int t dv$$

$$\int 3te^{2t} = ut - \int v du$$

$$\int 3te^{2t} = \frac{3t \cdot e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3 dt$$

$$= \frac{3te^{2t}}{2} - \int \frac{3e^{2t}}{2} dt$$

$$= \frac{3te^{2t}}{2} - \frac{3}{2} \int e^{2t} dt$$

$$= \frac{3te^{2t}}{2} - \frac{3}{2} \left[\frac{1 \cdot e^{2t}}{2} \right] + C$$

$$= \frac{3te^{2t}}{2} - \frac{3}{4} e^{2t} + C$$

$$= \frac{3e^{2t}}{2} \left[t - \frac{1}{2} \right] + C$$

c $x^2 \sin x$

Soln

$$\text{let } u = x^2$$

$$dv = \sin x$$

$$\frac{du}{dx} = 2x$$

$$\int dv = \int \sin x$$

$$v = -\cos x$$

$$du = 2x dx$$

$$\therefore \int x^2 \sin x = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x dx$$

$$= -x^2 \cos x + \int 2x \cos x dx$$

$$= -x^2 \cos x + 2 \int x \cos x dx$$

$$= -x^2 \cos x + 2 \left[\begin{array}{l} u = x \quad dv = \cos x \\ \frac{du}{dx} = 1 \quad \int dv = \sin x \\ du = dx \quad v = \sin x \end{array} \right]$$

$$= -x^2 \cos x + 2 \left[x \sin x - \int \sin x dx \right]$$

$$= -x^2 \cos x + 2 \left[x \sin x - (-\cos x) \right] + C$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

d $\cos 5x \cos 6x$

$$A = 5x, B = 6x$$

$$\int \cos A \cos B = \int \frac{1}{2} (\cos(A+B) + \cos(A-B))$$

$$\int \cos 5x \cos 6x = \int \frac{1}{2} (\cos(11x) + \cos(-x))$$

$$\int \cos 5x \cos 6x = \frac{1}{2} \int \cos 11x + \cos(-x)$$

$$= \frac{1}{2} \left[\frac{\sin 11x}{11} + \left(-\frac{\sin(-x)}{1} \right) \right] + C$$

$$= \frac{1}{2} \left[\frac{\sin 11x}{11} - \sin(-x) \right] + C$$

$$= \frac{\sin 11x}{22} - \frac{\sin(-x)}{2} + C$$

5 $\int \sin 7x \cos 2x$

let $A = 7x$, $B = 2x$

$$\int \sin A \cos B = \int \frac{1}{2} \sin(A+B) + \sin(A-B)$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \int \sin 9x + \sin 5x$$

$$= \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$