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Course | MAT 104
Department | Medicine and Surgery
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Assignment

1. $2x^2 \ln x$
2. $3te^{2t}$
3. $x^2 \sin x$
4. $\cos 5x \cos 6x$
5. $\sin 7x \cos 2x$

Solution

1. $2x^2 \ln x$

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{2x^3}{3}$

$du = \frac{dx}{x}$

$$\int u\,dv = uv - \int v\,du$$

$$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$$

$$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$$

$$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$$

$$\therefore \int 2x^2 \ln x\,dx = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$$

2. $3te^{2t}$

$u = 3t \qquad dv = e^{2t}$

$\frac{du}{dx} = 3 \qquad v = \frac{1}{2} e^{2t}$

$du = 3dx$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dx$

$= \frac{3t}{2}e^{2t} - \int \frac{3}{2}e^{2t}dx$

$= \frac{3te^{2t}}{2} - \frac{1}{2} \times \frac{3}{2}e^{2t} + C$

$\therefore \int 3te^{2t}dt = \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t} + C$

(3) $x^2 \sin x$

$u = \sin x \qquad dv = x^2$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx \qquad v = \frac{x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x\,dx = \sin x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \cos x\,dx$

$= \sin x \cdot \frac{x^3}{3} -$

$u = \cos x \quad dv = \frac{x^3}{3}$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx \quad v = \frac{x^4}{4}$

$x^2 \cos x - \int \frac{x^4}{4} \cdot -\sin x\,dx$

(3) $x^2 \sin x$

$u = x^2 \qquad dv = \sin x$

$\frac{du}{dx} = 2x$

$du = 2x\,dx \qquad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x\,dx = x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + \int 2x\cos x\,dx$

$= -x^2\cos x + \left[\begin{array}{l} u = 2x \quad dv = \cos x \\ \frac{du}{dx} = 2 \\ du = 2dx \quad v = \sin x \\ 2x\sin x - \int \sin x \cdot 2dx \\ 2x\sin x + 2\cos x + c \end{array} \right]$

$\int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

(4) $\cos 5x \cos 6x$

$A = 5x, \quad B = 6x$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$= \frac{1}{2}\left[\cos(5x+6x) + \cos(5x-6x)\right]$

$= \frac{1}{2}\left[\cos 11x + \cos x\right]$

$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\left[\frac{\sin 11x}{11} + \sin x\right] + c$

$\therefore \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + c$

(5) $\sin 7x \cos 2x$

$A = 7x, B = 2x$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$= \frac{1}{2}\left[\sin(7x+2x) + \sin(7x-2x)\right]$

$= \frac{1}{2}\left[\sin 9x + \sin 5x\right]$

$\int \sin 7x \cos 2x = \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$\therefore \int \sin 7x \cos 2x = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + c$