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MATH 104

Integrate the following functions

1 $2x^2 \ln x \, dx$

$u = \ln x$; $dv = 2x^2$, $du = \frac{1}{x} dx$: $v = \frac{2x^3}{3}$

Recall that:

$\int u\,dv = uv - \int v\,du$

$\Rightarrow = \left(\ln x \cdot \frac{2x^3}{3}\right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

$= \left(\ln x \cdot \frac{2x^3}{3}\right) - \int \frac{2x^2 dx}{3}$

$= \left(\ln x \cdot \frac{2x^3}{3}\right) - \frac{2x^3}{9} + c$

$= \frac{2}{3}x^3 \left(\ln x - \frac{1}{3}\right) + c$

2 $3te^{2t} dt$

$u = 3t$: $dv = e^{2t}$, $du = 3dx$, $v = e^{2t} = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$= 3t \cdot \frac{e^{2t}}{2} - \int \frac{1e^{2t}}{2} \cdot 3$

$= 3t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3$

$= \frac{3}{2}te^{2t} - \int \frac{e^{2t}}{2} \cdot 3$

$= \frac{3te^{2t}}{2} - \int \frac{3}{2} e^{2t}$

$= \frac{3te^{2t}}{2} - \left[\frac{3}{2}\int e^{2t}\right]$

$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + c$

3 $\int x^2 \sin x\,dx$

$u = x^2$; $dv = \sin x$, $du = 2x\,dx$; $v = \cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2 - \cos x - \int - \cos x \cdot 2x\,dx$

$= -x^2 \cos x + 2x \sin x + c$

4 $\int \cos 5x \cos 6x\,dx$

let $A = 5x$ and $B = 6x$

Recall that

$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

$= \frac{1}{2}\left(\cos(5x + 6x) + \cos(5x - 6x)\right)$

$= \frac{1}{2}\left(\cos 11x - \cos x\right)$

$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int \left(\cos 11x - \cos x\right) dx$

$= \frac{1}{2} \cdot \left(\frac{\sin 11x}{11} - \sin x\right)$

$= \frac{\sin 11x}{2x} - \frac{\sin x}{2} + c$

$$\Rightarrow \int \cos 5x \cos 6x\,dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + c$$

5 $\int \sin 7x \cos 2x\,dx$

let $A = 7x$ and $B = 2x$

Recall that

$$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$= \frac{1}{2}(\sin(7x+2x) + \sin(7x-2x))$$

$$= \frac{1}{2}(\sin 9x + \sin 5x)$$

$$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\int(\sin 9x + \sin 5x)\,dx$$

$$= \frac{1}{2}\int(\sin 9x + \sin 5x)\,dx$$

$$= \frac{1}{2}\left(-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right.$$

$$\frac{1}{2}\left(-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right.$$

$$= -\frac{\cos 7x}{18} - \frac{\cos 5x}{18} + c$$

$$\Rightarrow \int \sin 7x \cos 2x\,dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + c.$$