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Assignment Solution

1.) Integrate the following:

i) $\int 2x^2 \ln x$

Integrating by parts

let $u = \ln x$ and $dv = 2x^2$ ∴ $du/dx = 1/x$ and $v = x^3/3$

$\int u dv = uv - \int v du$

$\int \ln x = 2\left(\ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\right) dx$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3}\right) dx = 2\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right)$

$= \frac{2x^3 \ln x}{3} - \frac{2x^3}{9} dx$

$= \frac{3 \times 2x^3 \ln x - 2x^3}{9} dx = \frac{2x^3(3\ln x - 1)}{9} + c$

$= \frac{2x^3 \ln x - 1}{3} + c$

$\therefore \int 2x^2 \ln x = \frac{2x^3 \ln x - 1}{3} + c$

2.) $\int 3te^{2t}$

Integrating by parts:

let $u = 3t$; $dv = e^{2t}$; $du/dt = 3$; $v = \int e^{2t} = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$= \int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3\,dt$

$= \int 3te^{2t} = \left(\frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}\right) dt$

$= \int 3te^{2t} = \frac{3}{4}te^{2t} - \frac{1}{4}e^{2t} + c$

3.) $\int x^2 \sin x$

Integrating by parts:

let $u = x^2$, $dv = \sin x$, $dx = 2x\,dx$, $v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x = (x^2 \cdot -\cos x - \int -\cos x \cdot 2x)\,dx$

$= (-x^2 \cdot \cos x - \int -2x\cos x)\,dx \longrightarrow (-x^2 \cdot \cos x)\,dx$

Use integration by parts to solve the underlined ones

let $u = x$, $du = dx$, $dv = \cos x$, $v = \sin x$

$\int x^2 \sin x\,dx = [-x^2\cos x + 2\int x\cos x]\,dx$

$\int x^2 \sin x\,dx = [-x^2 \cos x + 2(x\sin x)\,dx$

$\int x^2 \sin x\,dx = -x^2 \cos x + 2x\sin x + C$

$\therefore \int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + c$

4.) $\int \cos 5x \cos 6x$

$A = 5x \quad , \quad B = 6x$

Recall: $\frac{1}{2}[\cos(A+B) + \cos(A-B)] = \cos A \cos B$

$= \frac{1}{2}\int [\cos(5x + 6x) + \cos(5x - 6x)]\, dx$

$= \frac{1}{2}\int [\cos 11x + \cos(-x)]\, dx$

$= \frac{1}{2}\int [\cos 11x - \cos x]\, dx$

$= \frac{1}{2}\left[-\frac{\sin 11x}{11} + \frac{\sin x}{1}\right] + c$

$= -\frac{\sin 11x}{22} + \frac{\sin x}{2} + c$

$\therefore \int \cos 5x \cos 6x = -\frac{\sin 11x}{22} + \frac{\sin x}{2} + c$

5.) $\int \sin 7x \cos 2x$; let $A = 7x$, $B = 2x$

Recall: $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{1}{2}\int[\sin(7x + 2x) + \sin(7x - 2x)]\, dx$

$= \frac{1}{2}\int[\sin 9x + \sin 5x]\, dx$

$= \frac{1}{2}\int - \cos 9x + (-\cos 5x)\, dx = \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$\therefore \int \sin 7x \cos 2x = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$