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DEPARTMENT: MBBS

MATRIC NUMBER: 19/MHS01/320

COURSE: MAT 104

### ASSIGNMENT

Integrate the following functions.

1.  $2x^2 \ln x$

Solution

$$\text{Let } u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

$$\frac{dv}{dx} = 2x^2 \quad \therefore dv = 2x^2 dx$$

$$v = \int 2x^2 = \frac{2x^3}{3}$$

$$\Rightarrow \int 2x^2 \ln x = 2 \int x^2 \ln x dx$$

$$\int u dv = uv - \int v du$$

$$\int 2x^2 \ln x dx = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$$

$$\int 2x^2 \ln x dx = 2 \left[ \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} \right] dx$$

$$\int 2x^2 \ln x dx = 2 \left[ \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} \right] dx$$

$$\int 2x^2 \ln x dx = 2 \left[ \frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 \right] dx$$

$$\int 2x^2 \ln x dx = 2 \left[ \frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3} \right] dx$$

$$\int 2x^2 \ln x dx = 2 \left[ \frac{x^3 \ln x}{3} - \frac{x^3}{9} \right] dx$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3 \ln x}{3} - \frac{2x^3}{9} \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{3 \times 2x^3 \ln x - 2x^3}{9} \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3(3 \ln x - 1)}{9} + C$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3(3 \ln x - 1)}{9} + C$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3(\ln x - 1)}{3} + C$$

$$\therefore \int 2x^2 \ln x \, dx = \frac{2x^3(\ln x - 1)}{3} + C //$$

2.  $3te^{2t}$

Solution: Integrating by parts.

Let  $u = 3t$

$$\frac{du}{dt} = 3$$

$$du = 3dt$$

$$du = 3dt$$

$$\frac{dv}{dt} = e^{2t} \quad \therefore dv = e^{2t} dt$$

$$v = \int e^{2t} = \frac{1}{2} e^{2t}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int 3te^{2t} dt = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 dt \therefore$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \int \frac{3e^{2t}}{2} dt$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \frac{1}{2} \cdot \frac{3}{2} e^{2t} + C$$

$$\int 3te^{2t} dt = \left[ \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] dt$$

$$\int 3te^{2t} dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

$$\therefore \int 3te^{2t} dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C //$$

3.  $x^2 \sin x$

Solution.

$$\text{Let } u = x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$du = 2x dx$$

$$\frac{dv}{dx} = \sin x \quad \therefore dv = \sin x dx$$

$$v = \int \sin x = -\cos x$$

$$\int u dv = uv - \int v du$$

$$\int x^2 \sin x dx = x^2 \cdot (-\cos x) - \int (-\cos x) \cdot 2x dx$$

$$\int x^2 \sin x dx = (-x^2 \cos x - \int 2x \cos x dx)$$

$$\int x^2 \sin x dx = (-x^2 \cos x + 2 \int x \cos x dx)$$

Using integration by parts to solve the underlined area.

$$\text{Let } u = x$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\frac{dv}{dx} = \cos x \quad \therefore dv = \cos x dx$$

$$v = \sin x$$

$$\int x^2 \sin x dx = [-x^2 \cos x + 2 \int x \cos x dx]$$

$$\int x^2 \sin x dx = [-x^2 \cos x + 2(x \sin x)] dx$$

$$\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + C$$

$$\therefore \int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + C$$

4.  $\cos 5x \cos 6x$

Solution

$$A = 5x, \quad B = 6x$$

Recall that;

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int [\cos(5x+6x) + \cos(5x-6x)] dx$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int [\cos 11x + \cos(-x)] dx$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int [\cos 11x - \cos x] dx$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \left[ \frac{-\sin 11x}{11} + \frac{\sin x}{1} \right] + C$$

$$\int \cos 5x \cos 6x dx = -\frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

$$\therefore \int \cos 5x \cos 6x dx = -\frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

5.  $\sin 7x \cos 2x$

Solution

$$A = 7x, \quad B = 2x$$

Recall that;

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} \int [\sin(7x+2x) + \sin(7x-2x)] dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2} \int [\sin 9x + \sin 5x] \, dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2} \int [-\cos 9x + (-\cos 5x)] \, dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2} \left[ -\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C //$$