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Question.

- Integrate the following functions:

1) $2x^2 \ln x$.
2) $3t e^{2t}$.
3) $x^2 \sin x$.
4) $\cos 5x \cos 6x$.
5) $\sin 7x \cos 2x$.

Solutions.

1) $\int 2x^2 \ln x$.

Let $u = \ln x$, $dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x}$, $v = \frac{2x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$\int 2x^2 \ln x = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^{\cancel{3}2}}{3} \cdot \frac{1}{\cancel{x}}\,dx$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3}\,dx$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} \ln x - \frac{2}{3}\left(\frac{x^3}{3}\right) + C$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C \quad \text{OR}$$

$$\int 2x^2 \ln x = \frac{2x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$$

2.) $\int 3te^{2t}dx \Rightarrow 3\int te^{2t}dx$

Let $u = t$, $dv = e^{2t}$

$\frac{du}{dt} = 1$, $v = \frac{e^{2t}}{2}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t}dx = t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 1$

$\int 3te^{2t}dx = \frac{te^{2t}}{2} - \frac{1}{2}\int e^{2t}$

$\int 3te^{2t}dx = 3\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right) + c$

$\int 3te^{2t}dx = \left(\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}\right) + c$

$$\left(\frac{}{2} \qquad \frac{}{4} \right) + C$$

3) $\int x^2 \sin x \, dx$

Let $u = x^2$, $dv = \sin x$

$\frac{du}{dx} = 2x\,dx$, $V = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \sin x \, dx = x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$

$\int x^2 \sin x \, dx = -x^2 \cos x + \int \cos x \cdot 2x\,dx$

$u = 2x$, $dv = \cos x$

$\frac{du}{dx} = 2\,dx$, $V = \sin x$

$\int u\,dv = uv - \int v\,du$

$\int 2x \cos x\,dx = 2x \cdot \sin x - \int \sin x \cdot 2\,dx$

$\int 2x \cos x\,dx = 2x \sin x + 2\cos x + C$

$\int x^2 \sin x\,dx = -x^2 \cos x + 2x \sin x + 2\cos x + C.$

4.) $\int \cos 5x \cos 6x \, dx$.

Let $A = 5x$, $B = 6x$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\left[\cos 11x - \cos x\right]$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\int (\cos 11x - \cos x)$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\left[\frac{\sin 11x}{11} - \sin x\right] + c$$

$$\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C.$$

5) $\int \sin 7x \cos 2x \, dx$

Let $A = 7x$, $B = 2x$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\sin 9x + \sin 5x\right]$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int (\sin 9x + \sin 5x)$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$\int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10}$