

① $2x^3 \ln x$

$u = \ln x \quad dv = 2x^3$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{2x^4}{4} = \frac{x^4}{2}$

$\int u dv = uv - \int v du$

$= \ln x \cdot \frac{2x^4}{4} - \int \frac{x^4}{2} \cdot \frac{1}{x} dx$

$= \frac{2x^4}{4} \ln x - \int \frac{x^3}{2} dx$

$= \frac{2x^4 \ln x}{4} - \frac{2x^4}{8} + c$

$\therefore \int 2x^3 \ln x dx = \frac{x^4 \ln x}{2} - \frac{x^4}{4} + c$

② $3te^{2t}$

$u = 3t \quad dv = e^{2t}$

$du = 3 dt \quad v = \frac{1}{2} e^{2t}$

$\int u dv = uv - \int v du$

$\int 3te^{2t} = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 dt$

$\int 3te^{2t} = \frac{3t}{2} e^{2t} - \int \frac{3}{2} e^{2t} dt$

$= \frac{3t}{2} e^{2t} - \frac{1}{2} \cdot \frac{3}{2} e^{2t} + c$

$\therefore \int 3te^{2t} dt = \left[\frac{3t}{2} e^{2t} - \frac{3}{4} e^{2t} \right] + c$

$x^2 \sin x \quad y = x^2 \quad dv = \sin x \quad \frac{dy}{dx} = 2x \quad du = 2x dx$

$\int x^2 \sin x dx = x^2 \cos x - \int \cos x \cdot 2x dx$

$= x^2 \cos x + \int 2x \cos x dx$

$-x^2 \cos x + \left[\begin{matrix} u = 2x & dv = \cos x \\ du = 2 dx & v = \sin x \end{matrix} \right]$

$= x^2 \cos x + 2x \sin x - \int 2 \sin x dx$

$= x^2 \cos x + 2x \sin x + \cos x + c$

$\therefore \int x^2 \sin x dx = x^2 \cos x + 2x \sin x + \cos x + c$

$$4) \cos 5x \cos 6x$$

$$A = 5x \quad B = 6x$$

Recall that: $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

$$= \frac{1}{2} [\cos 11x + \cos x]$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int [\frac{\sin 11x}{11} + \sin x] dx + C$$

$$\therefore \int \cos 5x \cos 6x dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

$$5) \sin 7x \cos 2x$$

$$A = 7x \quad B = 2x$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$\therefore \int \sin 7x \cos 2x dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$