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1. Find $\int 2x^2 \ln x \, dx$

Soln.

$= 2\int \ln x \cdot x^2 dx$ Let $u = \ln x$ $dv = x^2$ [Using integration by part]

$du = \frac{1}{x} dx$ $v = \frac{x^3}{3}$

Recall $\int u \, dv = uv - \int v \, du$ [Integration by part]

$2\int \ln x \cdot x^2 dx = 2\left(\ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx\right)$

$2\int \ln x \cdot x^2 dx = 2\left(\ln x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2 dx\right) = 2\left(\ln x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3}\right)$

$= \frac{2x^3}{3}\left[\ln x - \frac{1}{3}\right] + C$

2. Find $\int 3te^{2t} dt$

Soln.

Let $3\int te^{2t} dt = \int u \, dv$ $\therefore u = t$ $dv = e^{2t}$

$du = 1 \, dt$ $v = \frac{e^{2t}}{2}$

$\therefore 3\int te^{2t} dt = uv - \int v \, du$

$3\int te^{2t} dt = 3\left(t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot dt\right) = 3\left(t \cdot \frac{e^{2t}}{2} - \frac{1}{2}\int e^{2t} dt\right)$

$3\int te^{2t} dt = 3\left(t \cdot \frac{e^{2t}}{2} - \frac{1}{2} \cdot \frac{1}{2} e^{2t}\right) = \frac{3e^{2t}}{2}\left[t - \frac{1}{2}\right]$

Find

3. $\int x^2 \sin x \, dx$

Soln

Let $\int x^2 \sin x \, dx = \int u \, dv \quad \therefore u = x^2 \qquad \sin x \, dx = dv$

$du = 2x \, dx \qquad v = -\cos x$

Recall: $\int u \, dv = uv - \int v \, du$ [Integration by parts]

$\therefore \int x^2 \sin x \, dx = x^2 . (-\cos x) - 2\int x . -\cos x \, dx$

$" \quad = -x^2 \cos x + 2\int x \cos x \, dx$

Using integration by parts for $\int x \cos x \, dx$; $u = x$, $dv = \cos x \, dx$

$du = dx \quad v = \sin x$

$= -x^2 \cos x + 2\left[x . \sin x - \int \sin x \, dx \right]$

$\left[\text{if } \int x \cos x \, dx = \int u \, dv; \int x \cos x \, dx = uv - \int v \, du \right]$

$= -x^2 \cos x + 2\left[x \sin x - (-\cos x) \right]$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C_u$

4. Find $\int \cos 5x \cos 6x \, dx$

Solnu

Recall, $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

~~Let A = 5x~~

$= \frac{1}{2}\int 2\cos 5x \cos 6x \, dx$

Let $A = 5x$; $B = 6x$

$$= \frac{1}{2}\int [\cos 11x + \cos(-x)]\,dx$$

$$= \frac{1}{2}\int \cos 11x\,dx + \frac{1}{2}\int \cos(-x)\,dx$$

recall, $\cos(-x) = \cos x$

$$= \frac{1}{2} \cdot \frac{\sin 11x}{11} + \frac{1}{2} \cdot \sin x + C$$

$$= \frac{\sin(x)}{2} + \frac{\sin(11x)}{22} + C_{u}$$

5. Find $\int \sin 7x \cos 2x\,dx$

Soln

Recall; $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$\therefore \sin A \cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$

$$= \int \frac{1}{2}\sin 9x + \frac{1}{2}\sin 5x\,dx \quad \left[A = 7x, B = 2x\right]$$

$$= \frac{1}{2}\int \sin 9x\,dx + \frac{1}{2}\int \sin 5x\,dx$$

$$= \frac{1}{2} \cdot \frac{-\cos 9x}{9} + \frac{1}{2} \cdot \frac{-\cos 5x}{5} + C$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C_{u}$$