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1) $2x^2 \ln x$

$$\int 2x^2 \ln(x)\, dx$$

$$2\int x^2 \ln(x)\, dx$$

Integral by parts: $\int fg' = fg - \int f'g$

$f = \ln(x)$, $g' = x^2$

$f' = \frac{1}{x}$ $g = \frac{x^3}{3}$

$$\int fg' = \frac{x^{2+1}\ln(x)}{2+1} - \int \frac{1}{x} \cdot \frac{x^{2+1}}{3}\, dx$$

$$= \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3}\, dx$$

$$\frac{1}{3}\int x^2\, dx$$

$\therefore \cancel{\frac{1}{3}\int \frac{x^3}{9}dx}$

$\therefore \frac{1}{3} \cdot \frac{x^3}{9}$

$\cancel{\int} = \frac{x^3 \ln(x)}{3} - \frac{x^3}{9}$

$2\int x^2 \ln(x)\,dx = 2\left(\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right)$

$\frac{2x^3 \ln(x)}{3} - \frac{2x^3}{9} + C$

$\therefore \frac{2x^3 (3\ln(x) - 1)}{9} + C$

(2) $3t\,e^{2t}$

$3\int t e^{2t}\,dt$

$\int t e^{2t}$

Integral by parts: $\int fg' = fg - \int f'g$

$f = t \quad g' = e^{2t}$

$f' = 1 \quad g = \frac{e^{2t}}{2}$

$= \frac{te^{2t}}{2} - \int \frac{e^{2t}dt}{2}$

$\downarrow$

$\int \frac{e^{2t}dt}{2}$

let $u = 2t$

$\frac{du}{dt} = 2$

$\therefore dt = \frac{1}{2}du$

$\therefore \frac{1}{4}\int e^{u}du$

$\downarrow$

$\int e^{u}du$

Using exponential rule:

$\int e^{u}du = \frac{a^{u}}{\ln(a)}$

$a = e$

$\therefore \int e^{u}du = e^{u}$

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$$\frac{1}{4}\int e^{u}\,du$$

$$= \frac{e^{u}}{4}$$

$$= \frac{e^{2t}}{4}$$

$$\therefore \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt$$

$$= \frac{te^{2t}}{2} - \frac{e^{2t}}{4}$$

$$3\int te^{2t}\,dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

$$= \frac{3(2t-1)\,e^{2t}}{4} + C$$

$$\therefore = \frac{(6t-3)\,e^{t}}{4} + C$$

(3) $x^2 \sin x$

$$\int x^2 \sin x =$$

Integral by parts: $\int fg' = fg - \int f'g$

$f = x^2$, $g' = \sin(x)$

$f' = 2x$ $\quad g = -\cos(x)$

$$\int fg' = -x^2\cos(x) - \int -2x\cos(x)\,dx$$

$$\int -2x\cos(x)\,dx = 2\int x\cos(x)\,dx$$

Integral by parts: $\int fg' = fg - \int f'g$

$f = x$, $g' = \cos(x)$

$f' = 1$ $\quad g = \sin(x)$

$$\int fg' = x\sin(x) - \int \sin(x)\,dx$$

$$\int \sin(x) = -\cos(x)$$

$$\int fg' = x\sin(x) + \cos(x)$$

$$\therefore \int -2x\cos(x)\,dx = -2x\sin(x) + 2\cos(x)$$

$$\int x^2 \sin(x) = 2x\sin(x) - x^2\cos(x) + 2\cos(x) + C$$

$$= 2x\sin(x) + (2-x^2)\cos(x) + C$$

4) $\cos 5x \cos 6x$

From product-to-sum formula:

$$\cos(x)\cos(y) = \frac{1}{2}\left(\cos(y+x) + \cos(y-x)\right)$$

$$\therefore \int \cos 5x \cos 6x = \int \frac{1}{2}\cos(6x+5x) + \frac{1}{2}\cos(6x-5x)$$

$$= \frac{1}{2}\int \cos(11x)\,dx + \frac{1}{2}\int \cos(x)\,dx$$

$$\int \cos(11x)\,dx$$

let $u = 11x$ $\qquad \frac{du}{dx} = 11$

$$dx = \frac{1}{11}du$$

$$\frac{1}{11}\int \cos u \, du$$

$$\int \cos(u)\,du = \sin(u)$$

$$\therefore \frac{1}{11}\int \cos(u)\,du = \frac{\sin(u)}{11}$$

$$= \frac{\sin(11x)}{11}$$

$$\frac{1}{2}\int \cos(x)\,dx = \frac{1}{2}\sin x$$

$$= \frac{\sin(11x)}{22} + \frac{\sin(x)}{2}$$

$$\frac{\sin(11x) + 11\sin(x)}{22} + C$$

(5) $\sin 7x \cos 2x$

$$\int \sin 7x \cos 2x = \int \frac{\sin(9x) + \sin(5x)}{2}\,dx$$

$$\frac{1}{2}\int \sin(9x)\,dx + \frac{1}{2}\int \sin(5x)\,dx$$

$$\int \sin(9x)\,dx$$

$$u = 9x \qquad \frac{du}{dx} = 9$$

$$dx = \frac{1}{9}\,du$$

$$\frac{1}{2}\times\frac{1}{9}\int \sin(u)\,du$$

$$\frac{1}{2}\times\frac{1}{9}\int \sin(u)\,du = \cancel{\sin(9x)} \quad -\frac{\cos u}{9}$$

$$\therefore = -\frac{\cos(9x)}{18}$$

$$\frac{1}{2}\int \sin(5x)\,dx \qquad u = 5x \qquad \frac{du}{dx} = 5$$

$$dx = \frac{1}{5}\,du$$

$$\frac{1}{5}\times\frac{1}{2}\int \frac{\sin(u)}{\cancel{\cos u}}\,du = \frac{1}{2}\times -\frac{\cos(u)}{5}$$

$$= \frac{1}{10}\times -\cos(5x)$$

$$= -\frac{\cos 5x}{10}$$

$$\therefore \frac{1}{2}\int \sin(9x)\,dx + \frac{1}{2}\int \sin(5x)\,dx =$$

$$= \frac{-\cos 9x}{18} - \frac{\cos(5x)}{10} + c$$