Name: [illegible]
Matric No: 19/MHS01/043
Department: Medicine And Surgery
Course: MTH 104
Assignment
(1) Integrate the following functions,
(a) $2x^2 \ln x \, dx$

Solution

$u = \ln x \quad ; \quad dv = 2x^2$
$du = \frac{1}{x} dx \quad ; \quad v = \frac{2x^3}{3}$

Recall that
$$\int u \, dv = uv - \int v \, du$$
$$\Rightarrow \left(\ln x \cdot \frac{2x^3}{3} \right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} \, dx$$
$$= \left(\ln x \cdot \frac{2x^3}{3} \right) - \int \frac{2x^3}{3} \, dx$$
$$= \left(\ln x \cdot \frac{2x^3}{3} \right) - \frac{2x^3}{9} + c$$
$\therefore$ Integral of
$$2x^2 \ln x \, dx = \frac{2x^3}{3} \left(\ln x - \frac{1}{3} \right) + c$$

(b) $3te^{2t} \, dt$
$U = 3t \quad ; \quad dv = e^{2t}$
$du = 3dx \quad ; \quad v = e^{2t} = \frac{1}{2} e^{2t}$
$$\int U \, dv = Uv - \int v \, du$$
$$= 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3$$
$$= 3t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 3$$
$$= \frac{3}{2} te^{2t} - \int \frac{e^{2t}}{2} \cdot 3$$
$$= \frac{3}{2} te^{2t} - \int \frac{3}{2} e^{2t}$$
$$= \frac{3}{2} te^{2t} - \left[\frac{3}{2} \int e^{2t} \right]$$
$\therefore$ Integral of $3te^{2t} \, dt = \frac{3}{2} te^{2t} - \left[\frac{3}{2} \int e^{2t} \right]$

(c) $\int x^2 \sin x \, dx$

$u = x^2 \quad dv = \sin x$

$du = 2x\,dx \; ; \; v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2 \cos x + 2x \sin x + c$

$\therefore$ Integral of $\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + c$

(d) $\int \cos 5x \cos 6x \, dx$

Let $A = 5x$ and $B = 6x$

Recall that

$$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$= \frac{1}{2}\left(\cos(5x+6x) + \cos(5x-6x)\right)$$

$$= \frac{1}{2}\left(\cos 11x - \cos x\right)$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2}\int\left(\cos 11x - \cos x\right) dx$$

$$\Rightarrow \frac{1}{2} \cdot \left(\frac{\sin 11x}{11} - \sin x\right)$$

$$= \frac{\sin 11x}{22} - \frac{\sin x}{2} + c$$

$\therefore$ Integral of $\int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + c$

(e) $\int \sin 7x \cos 2x \, dx$

Let $A = 7x$ and $B = 2x$

Recall that

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$= \frac{1}{2}\left(\sin(7x+2x) + \sin(7x-2x)\right)$$

$$= \frac{1}{2}\left(\sin 9x + \sin 5x\right)$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int\left(\sin 9x + \sin 5x\right) dx$$

$$= \frac{1}{2}\left(\sin(7x + 2x) + \sin(7x - 2x)\right)$$

$$= \frac{1}{2}\left(\sin 9x + \sin 5x\right)$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int \left(\sin 9x + \sin 5x\right) dx$$

$$= \frac{1}{2}\int \left(\sin 9x + \sin 5x\right) dx$$

$$= \frac{1}{2} \cdot \left(\frac{-\cos 9x}{9} - \frac{\cos 5x}{5}\right)$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + c$$

$$\Rightarrow \int \sin 7x \cos 2x \, dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$