

$$1. \int 2x^2 \ln x \, dx$$

Solution

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}, \quad dx = \frac{1}{x} \, dx$$

$$dv = 2x^2 \, dx$$

$$\int dv = \int 2x^2 \, dx$$

$$v = \frac{2x^3}{3}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \int x^2 \, dx$$

$$= \frac{2}{3} x^3 \ln x - \frac{2}{3} \cdot \frac{x^3}{3} + C$$

$$= \frac{2}{3} x^3 \ln x - \frac{2x^3}{9} + C$$

$$\therefore \int 2x^2 \ln x \, dx = \frac{2}{3} x^3 \left(\ln x - \frac{1}{3} \right) + C$$

$$2. \int 3t e^{2t} \, dt$$

Solution

$$u = 3t$$

$$\frac{du}{dt} = 3, \quad dt = \frac{1}{3} \, du$$

$$dv = e^{2t} \, dt$$

$$v = \frac{e^{2t}}{2}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int 3t e^{2t} \, dt = \frac{3t}{2} (e^{2t}) - \frac{3}{2} \int e^{2t} \, dt$$

$$= \frac{3t}{2} (e^{2t}) - \frac{3}{2} \frac{e^{2t}}{2} + C$$

$$\int 3t e^{2t} \, dt = \frac{3t}{2} (e^{2t}) - \frac{3}{4} e^{2t} + C$$

$$\therefore \int 3t e^{2t} \, dt = \frac{3}{2} e^{2t} \left(t - \frac{1}{2} \right) + C$$

$$3. \int x^2 \sin x \, dx$$

Solution

$$u = x^2$$

$$du = 2x \, dx$$

$$dv = \sin x \, dx$$

$$v = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int (-\cos x)(2x \, dx)$$

$$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx$$

$$\text{but } \int x \cos x \, dx = ?$$

$$\int x \cos x \, dx$$

$u = x$
 $du = dx$
 $dv = \cos x \, dx$
 $v = \sin x$

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx$$

$$= x \sin x - (-\cos x)$$

$$= x \sin x + \cos x$$

$$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2(x \sin x + \cos x) + C$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4 \int \cos 5x \cos 6x \, dx$$

Solution

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$A = 5x, \quad B = 6x$$

$$\cos 5x \cos 6x = \frac{1}{2} [\cos(5x+6x) + \cos(5x-6x)]$$

$$= \frac{1}{2} [\cos 11x + \cos(-x)]$$

$$\cos(-x) = \cos x$$

$$= \frac{1}{2} [\cos 11x + \cos x]$$

$$\int \cos 5x \cos 6x \, dx = \frac{1}{2} \int (\cos 11x + \cos x) \, dx$$

$$= \frac{1}{2} \left(\frac{\sin 11x}{11} + \frac{\sin x}{1} \right) + C$$

$$\therefore \int \cos 5x \cos 6x \, dx = \frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

$$5 \int \sin 7x \cos 2x \, dx$$

Solution

$$\sin 7x \cos 2x = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2} \int [\sin 9x + \sin 5x] \, dx$$

$$= \frac{1}{2} \left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$\therefore \int \sin 7x \cos 2x \, dx =$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$