1. $2x^2 \ln x$

Solution

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$\frac{dv}{dx} = 2x^2 \quad \therefore dv = 2x^2 dx$

$v = \int 2x^2 = \frac{2x^3}{3}$

$\Rightarrow \int 2x^2 \ln x = 2\int x^2 \ln x \, dx$

$\int u \, dv = uv - \int v \, du$

$\int 2x^2 \ln x \, dx = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{dx}{x}$

$\int 2x^2 \ln x \, dx = 2\left[\frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x}\right] dx$

$\int 2x^2 \ln x \, dx = 2\left[\frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\right] dx$

$\int 2x^2 \ln x \, dx = 2\left[\frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\right] dx$

$\int 2x^2 \ln x \, dx = 2\left[\frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3}\right] dx$

$\int 2x^2 \ln x \, dx = 2\left[\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right] dx$

$\int 2x^2 \ln x\, dx = \frac{2x^3 \ln x}{3} - \frac{2x^3}{9}\, dx$

$\int 2x^2 \ln x\, dx = \frac{3 \times 2x^3 \ln x - 2x^3}{9}\, dx$

$\int 2x^2 \ln x\, dx = \frac{2x^3(3\ln x - 1)}{9} + C$

$\int 2x^2 \ln x\, dx = \frac{2x^3(3\ln x - 1)}{9} + C$

$\int 2x^2 \ln x\, dx = \frac{2x^3(\ln x - 1)}{3} + C$

$\therefore \int 2x^2 \ln x\, dx = \frac{2x^3(\ln x - 1)}{3} + C$ //

2. $3te^{2t}$

Solution: Integrating by parts.

Let $u = 3t$ $\qquad \frac{dv}{dt} = e^{2t} \therefore dv = e^{2t}dt$

$\frac{du}{dt} = 3$

$du = 3dt$ $\qquad v = \int e^{2t} = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t}dt = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dt$

$\int 3te^{2t}dt = \frac{3te^{2t}}{2} - \int \frac{3}{2}e^{2t} \cdot dt$

$$\int 3te^{2t}\,dt = \frac{3te^{2t}}{2} - \frac{1}{2}\cdot\frac{3}{2}e^{2t} + C$$

$$\int 3te^{2t}\,dt = \left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}\right]dt$$

$$\int 3te^{2t}\,dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

$$\therefore\ 3te^{2t}\,dt = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C \quad //$$

3. $x^2 \sin x$

Solution.

Let $u = x^2$ $\qquad \frac{dv}{dx} = \sin x \ \therefore\ dv = \sin x\,dx$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$ $\qquad V = \int \sin x = -\cos x$

$$\int u\,dv = uv - \int v\,du$$
$$\int x^2 \sin x\,dx = x^2 \cdot -\cos x - \int -\cos x \cdot 2x\,dx$$
$$\int x^2 \sin x\,dx = (-x^2\cos x - \int -2x\cos x)\,dx$$
$$\int x^2 \sin x\,dx = (-x^2\cos x + \underline{2\int x\cos x})\,dx$$

Using integration by parts to solve the underlined area.

Let $u = x$ $\qquad \frac{dv}{dx} = \cos x \ \therefore\ dv = \cos x\,dx$

$\frac{du}{dx} = 1$ $\qquad V = \sin x$

$$\int x^2 \sin x\,dx = [-x^2\cos x + 2\int x\cos x]\,dx$$
$$\int x^2 \sin x\,dx = [-x^2\cos x + 2(x\sin x)]\,dx$$
$$\int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + C$$
$$\therefore \int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + C$$

4. $\cos 5x \cos 6x$

Solution

$A = 5x \quad , \quad B = 6x$

Recall that;

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int [\cos(5x+6x) + \cos(5x-6x)]\,dx$$

$$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int [\cos 11x + \cos(-x)]\,dx$$

$$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\int [\cos 11x - \cos x]\,dx$$

$$\int \cos 5x \cos 6x\,dx = \frac{1}{2}\left[-\frac{\sin 11x}{11} + \frac{\sin x}{1}\right] + C$$

$$\int \cos 5x \cos 6x\,dx = -\frac{\sin 11x}{22} + \frac{\sin x}{2} + C$$

$$\therefore \int \cos 5x \cos 6x\,dx = -\frac{\sin 11x}{22} + \frac{\sin x}{2} + C \quad //$$

5. $\sin 7x \cos 2x$

Solution

$A = 7x \quad , \quad B = 2x$

Recall that;

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\int \sin 7x \cos 2x\,dx = \frac{1}{2}\int [\sin(7x+2x) + \sin(7x-2x)]$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int [\sin 9x + \sin 5x] \, dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int [-\cos 9x + (-\cos 5x)] \, dx$$

$$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right] + C$$

$$\therefore \int \sin 7x \cos 2x \, dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C \quad //$$