

SOLUTIONS

$$\int 2x^2 \ln x \, dx$$

$$u = \ln x \quad dv = 2x^2$$

$$du = \frac{1}{x} dx \quad v = \frac{2x^3}{3}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int u \, dv = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$$

$$\int u \, dv = \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$$

$$\int u \, dv = \frac{2x^3}{3} \ln x - \frac{2}{3} \int x^2 dx$$

$$\int u \, dv = \frac{2x^3}{3} \ln x - \frac{2}{3} \cdot \frac{x^3}{3} dx$$

$$\int u \, dv = \frac{2x^3}{3} \left[\ln x - \frac{1}{3} \right] + c$$

$$\int 3t e^{2t} dt$$

$$u = 3t, \quad dv = e^{2t}$$

$$du = 3 dt \quad v = \frac{1}{2} e^{2t}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int u \, dv = \frac{3t}{2} \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 dt$$

$$\int u \, dv = \frac{3}{2} t e^{2t} - \frac{3}{2} \int e^{2t} dt$$

$$\int u \, dv = \frac{3}{2} t e^{2t} - \frac{3}{2} \cdot \frac{1}{2} e^{2t} + c$$

$$\int u dv = \frac{3}{2} te^{2t} - \frac{3t^2}{4} + C$$

$$3 \int x^2 \sin x dx$$

$$u = x^2, \quad dv = \sin x$$

$$du = 2x dx, \quad v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$= x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x dx$$

$$= -\cos x \cdot x^2 + \int \cos x \cdot 2x dx \quad \text{--- (1)}$$

$$\text{from } \int \cos x \cdot 2x dx$$

$$u = 2x \quad dv = \cos x$$

$$du = 2 dx \quad v = \sin x$$

$$\Rightarrow 2x \sin x - \int \sin x \cdot 2 dx \quad \text{--- (2)}$$

$$\text{Eqn (2) to (1)}$$

$$= -x^2 \cos x + 2x \sin x - 2 \int \sin x dx$$

$$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4 \int \cos 5x \cos 6x$$

$$\text{Recall } \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\int \cos 5x \cos 6x = \frac{1}{2} [\cos(5x+6x) + \cos(5x-6x)]$$

$$\int \cos 5x \cos 6x = \frac{1}{2} [\cos(11x) + \cos(-x)]$$

$$= \frac{1}{2} \left[\frac{\sin 11x}{11} + \frac{\sin -x}{-1} \right]$$

$$\int \cos 5x \cos 6x = \frac{\sin 11x}{22} + \left(\frac{\sin -x}{-2} \right) + C$$

$$5 \int \sin 7x \cos 2x$$

Recall $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$

$$\int \sin 7x \cos 2x = \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$= \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right]$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$