MAT 104 ASSIGNMENT

1. $\int 2x^2 \ln x$

solution

$u = \ln x$, $dv = 2x^2$

$du = \frac{1}{x} dx$ $v = \frac{2x^3}{3}$

$$\int 2x^2 \ln x = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{2x^3}{3} \ln x - \int \frac{2x^2}{3} dx$$

$$= \frac{2x^3}{3} \ln x - \frac{2}{3} \int x^2 dx$$

$$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + c$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} \left(\ln x - \frac{1}{3} \right) + c$$

2. $\int 3t e^{2t}$

solution

$u = 3t$, $dv = e^{2t}$

$du = 3dt$ $v = \frac{1}{2} e^{2t}$

$$\int 3t e^{2t} = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \, 3dt$$

$$= \frac{3}{2} t e^{2t} - \frac{3}{2} \int e^{2t} dt$$

$$= \frac{3}{2} t e^{2t} - \frac{3}{2} \times \frac{1}{2} e^{2t} + c$$

$$= \frac{3}{2} t e^{2t} - \frac{3}{4} e^{2t} + c$$

$$\int 3t e^{2t} = \frac{3}{2} \left(t - \frac{1}{2} \right) + c$$

3. $\int x^2 \sin x$

Solution

$u = x^2$, $dv = \sin x$

$du = 2x\,dx$ $v = -\cos x$

$\int x^2 \sin x = -x^2 \cos x - \int -2x \cos x\,dx$

$= -x^2 \cos x + \int 2x \cos x\,dx$

$= -x^2 \cos x + 2x \sin x - \int 2 \sin x\,dx$

$= -x^2 \cos x + 2x \sin x - (-2\cos x) + C$

$\int x^2 \sin x = -x^2 \cos x + 2x \sin x + 2\cos x + C$

4. $\int \cos 5x \cos 6x$

Solution

Recall;

$\cos Ax \cos Bx = \frac{1}{2}[\cos(A-B)x + \cos(A+B)x]$

$\int \cos 5x \cos 6x = \int \frac{1}{2}[\cos(5-6)x + \cos(5+6)x]$

$= \int \frac{1}{2}[\cos(-x) + \cos(11x)]$

$= \frac{1}{2}\int \cos(-x) + \cos(11x)$

$= \frac{1}{2}\int \cos(-x)\,dx + \frac{1}{2}\cos(11x)\,dx$

Recall;
$\cos(-x) = \cos x$

$= \frac{1}{2}\int \cos(x)\,dx + \frac{1}{2}\int \cos(11x)\,dx$

$= \frac{1}{2} \times \frac{\sin(x)}{1} + \frac{1}{2} \times \frac{\sin(11x)}{11} + C$

$= \frac{\sin(x)}{2} + \frac{\sin(11x)}{22} + C$

5. $\int \sin 7x \cos 2x$

Solution

Recall,

$$\sin Ax \cos Bx = \frac{1}{2}[\sin(A+B)x + \sin(A-B)x]$$

$$\int \sin 7x \cos 2x = \frac{1}{2}\int [\sin(9x) + \sin(5x)]$$

$$= \frac{1}{2}\int \sin(9x)dx + \frac{1}{2}\int \sin(5x)dx$$

$$= \frac{1}{2}\left(\frac{-\cos(9x)}{9}\right) + \frac{1}{2}\left(\frac{-\cos(5x)}{5}\right) + C$$

$$= -\frac{\cos(9x)}{18} + \left(-\frac{\cos(5x)}{10}\right) + C$$

$$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$