Name: Ilo Stephanie Chidimma
Course: ~~Aeronautics~~ 1st Engg.
Dept: Aeronautics.
College: Engineering.
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Find the integral of the following.

(a) $\sin 7x \cos 2x \, dx$

Solution.

$\int \sin 7x \cos 2x \, dx$; using identity we have

$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$

$\sin 9 \cos b = \frac{1}{2}(\sin(7x+2x) + \sin(7x-2x))$

$\sin a \cos b = \frac{1}{2}(\sin 9x + \sin 5x)$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\int \sin 9x + \frac{1}{2}\int \sin 5x$

$\int \sin 7x \cos 2x \, dx = \frac{1}{2}\left[-\frac{\cos 9x}{9}\right] + \frac{1}{2}\left[-\frac{\cos 5x}{5}\right]$

$\int \sin 7x \cos 2x \, dx = -\frac{1}{18}\cos 9x - \frac{1}{10}\cos 5x$

$\therefore \int \sin 7x \cos 2x \, dx = -\frac{1}{18}\cos 9x - \frac{1}{10}\cos 5x + c_{11}$

(b) $\cos 3x \cos x \, dx$

Solution

$\int \cos 3x \cos x \, dx = \int \cos a \cos b$

Using identity we have

$\int \cos a \cos b = \int \frac{1}{2}(\cos(a-b) + \cos(a+b)) \Big] \begin{matrix} a = 3x \\ b = x \end{matrix}$

$\int \cos 3x \cos x = \int \frac{1}{2}(\cos(3x-x) + \cos(3x+x))$

$\int \cos 3x \cos x = \int \frac{1}{2}(\cos 2x + \cos 4x)$

$\int (\cos 3x \cos x) = \int \frac{1}{2}\cos 2x + \int \frac{1}{2}\cos 4x$

$\int \cos 3x \cos x = 1/2 \int \cos 2x + 1/2 \cos 4x)$

$\int \cos 3x \cos x = \frac{1}{2}\left[\frac{\cos 2x}{2}\right] + \frac{1}{2}\left(\frac{\cos 4x}{4}\right]$

$\int \cos 3x \cos x = \frac{1}{4}[\cos 2x] + \frac{1}{8}\cos 4x$

$= \int \cos 3x \cos x = \frac{1}{4}\cos 2x + \frac{\cos 4x}{8} + c_u$

(3) $\frac{\cos x}{\sin^2 x} dx$

Solution

$\int \frac{\cos x}{\sin^2 x} dx$

let $u = \sin x \therefore du = \cos dx$

$\therefore \int \frac{du}{u^2} = -\frac{1}{u}$

$= -\frac{1}{\sin x} = -\cos ecant + x$ or $-\cos x_u$

(4) Double integral with limits from 1 to 2 from 0 to $3(9x^2y)\, dx\, dy$.

Solution.

$\int_1^2 \int_0^3 9x^2y\, dx\, dy = \int_1^2 \left[\int_0^3 9x^2y\, dx\right] dy$

treating y as a constant

$\int_1^2 \left[\frac{9x^3}{3} y\, dx\right] dy = \int_1^3 \left[3x^3 y\, dx\right] dy$

where $x = (0, 3)$

$\int_1^2 \left[3(0)^3 y + 3(3)^3 y\right] dy$

$\int_1^2 \int_0^3 \left[3 \times 27 y\right] dy$

$\int_1^2 \int_0^3 \left[81 y\right] dy$

$\int_1^2 81 y = \left[\frac{3^4 y^2}{2}\right]_1^2$

$$= \frac{3^4 y^2}{2} \Big|_1^2$$

when $y = (2, 1)$

$$= \frac{3^4 (2)^2}{2} - \frac{3^4 (1)^2}{2}$$

$$= \frac{324}{2} - \frac{81}{2}$$

$$= \frac{324 - 81}{2}$$

$$= \frac{243}{2}$$

$$= 121.5_4$$