

1. $\int 2x^2 \ln x$

solution

$$u = \ln x, \quad dv = 2x^2$$

$$du = \frac{1}{x} dx, \quad v = \frac{2x^3}{3}$$

$$\int 2x^2 \ln x = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{2x^3 \ln x}{3} - \int \frac{2x^2}{3} dx$$

$$= \frac{2x^3 \ln x}{3} - \frac{2}{3} \int x^2 dx$$

$$= \frac{2x^3 \ln x}{3} - \frac{2x^3}{9} + C$$

$$\int 2x^2 \ln x = \frac{2x^3}{3} \left(\ln x - \frac{1}{3} \right) + C$$

2. $\int 3te^{2t}$

solution

$$u = 3t, \quad dv = e^{2t}$$

$$du = 3dt, \quad v = \frac{1}{2}e^{2t}$$

$$\int 3te^{2t} = 3t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot 3dt$$

$$= \frac{3}{2}te^{2t} - \frac{3}{2} \int e^{2t} dt$$

$$= \frac{3}{2}te^{2t} - \frac{3}{2} \times \frac{1}{2}e^{2t} + C$$

$$= \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$$

$$\int 3te^{2t} = \frac{3}{2} \left(t - \frac{1}{2} \right) e^{2t} + C$$

$$3. \int x^2 \sin x$$

Solution

$$u = x^2$$

$$, dv = \sin x$$

$$du = 2x dx$$

$$v = -\cos x$$

$$\int x^2 \sin x = -x^2 \cos x - \int -2x \cos x dx$$

$$= -x^2 \cos x + \int 2x \cos x dx$$

$$= -x^2 \cos x + 2x \sin x - \int 2 \sin x dx$$

$$= -x^2 \cos x + 2x \sin x - (-2 \cos x) + C$$

$$\int x^2 \sin x = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4. \int \cos 5x \cos 6x$$

Solution

Recall,

$$\cos A x \cos B x = \frac{1}{2} [\cos(A-B)x + \cos(A+B)x]$$

$$\int \cos 5x \cos 6x = \int \frac{1}{2} [\cos(5-6)x + \cos(5+6)x]$$

$$= \int \frac{1}{2} [\cos(-x) + \cos(11x)]$$

$$= \frac{1}{2} \int \cos(-x) + \cos(11x)$$

$$= \frac{1}{2} \int \cos(-x) dx + \frac{1}{2} \int \cos(11x) dx$$

Recall,

$$\cos(-x) = \cos x$$

$$= \frac{1}{2} \int \cos(x) dx + \frac{1}{2} \int \cos(11x) dx$$

$$= \frac{1}{2} \times \frac{\sin(x)}{1} + \frac{1}{2} \times \frac{\sin(11x)}{11} + C$$

$$= \frac{\sin(x)}{2} + \frac{\sin(11x)}{22} + C$$

$$5. \int \sin 7x \cos 2x$$

Solution

~~Recall, $\int \sin A \cos B$~~

~~$\int \sin A \cos B dx$~~

Recall,

$$\sin A x \cos B x = \frac{1}{2} [\sin(A+B)x + \sin(A-B)x]$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \int [\sin(9x) + \sin(5x)]$$

$$= \frac{1}{2} \int \sin(9x) dx + \frac{1}{2} \int \sin(5x) dx$$

$$= \frac{1}{2} \left(\frac{-\cos(9x)}{9} \right) + \frac{1}{2} \left(\frac{-\cos(5x)}{5} \right) + C$$

$$= \frac{-\cos(9x)}{18} + \left(\frac{-\cos(5x)}{10} \right) + C$$

$$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$