ANUGWU FRANKLIN CHIEMELIE
19/MHS01/097
MBBS
MAT 104

Integrate the following functions
① $2x^2 \ln x$
② $3te^{2t}$
③ $x^2 \sin x$
④ $\cos 5x \cos 6x$
⑤ $\sin 7x \cos 2x$

Solution

① $\int 2x^2 \ln x \, dx$

let $u = \ln x$, $dv = 2x^2$

$du = \frac{1}{x} dx$, $v = \frac{2x^3}{3}$

$\int u dv = uv - \int v du$

$= \ln x \left(\frac{2x^3}{3}\right) - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$

then; $\frac{2x^3}{3} \ln x - \frac{2x^2}{3} dx$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9} + C$

② $\int 3te^{2t}$; let $u = 3t$, $dv = e^{2t}$, $du = 3dx$, $v = e^{2t}$

$\int u dv = uv - \int v du$

$= 3t(e^{2t}) - \int e^{2t} \cdot 3dx$

$= 3t(e^{2t}) - 3 \cdot 2e^{2t} + C$

③ $\int x^2 \sin x \, dx$

let $u = x^2$, $dv = \sin x$

$\frac{du}{dx} = 2x$, $V = \cos x$

$$\int u dv = uv - \int v du$$

$$= x^2(\cos x) - \int \cos x \, 2x dx$$

$$= x^2 \cos x - \int \cos x \left[\frac{du}{2x} \times \frac{2x}{1} \right]$$

$$\int u dv = x^2 \cos x - \cos x + C$$

④ $\int \cos 5x \cos 6x \, dx$

let $A = 5x$, $B = 6x$

$$\cos A \cos B = \frac{1}{2}\left[\cos(5x+6x) + \cos(5x-6x)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos 11x + \cos(-x)\right]$$

$$= \frac{1}{2}\left[\frac{\cos 11x}{11} + \frac{\cos(-x)}{1}\right]$$

$$\int \cos 5x \cos 6x \, dx = \frac{\cos 11x}{22} + \frac{\cos(-x)}{2} + C$$

⑤ $\int \sin 7x \cos 2x \, dx$

let $A = 7x$, $B = 2x$

$$\sin A \cos B = \frac{1}{2}\left[\sin 9x + \sin 5x\right]$$

$$= \frac{1}{2}\left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5}\right]$$

$$\int \sin 7x \cos 2x \, dx = \frac{\sin 9x}{18} + \frac{\sin 5x}{10} + C$$