

$$1. \int 2x^2 \ln x \, dx.$$

$$\text{Let } u = \ln x \quad dv = 2x^2$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$v = \frac{2x^3}{3}$$

$$du = \frac{1}{x} dx$$

$$\int u \, dv = uv - \int v \, du$$

$$\int 2x^2 \ln x \, dx = \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} \, dx.$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3}{3} \ln x - \frac{2}{3} \int x^2 \cdot \frac{1}{x} \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3}{3} \ln x - \frac{2}{3} \left[\frac{x^3}{3} \right] + C.$$

$$\int 2x^2 \ln x \, dx = \frac{2x^3}{3} \left[\ln x - \frac{1}{3} \right] + C$$

$$2. \int 3te^{2t} \, dt$$

$$\text{Let } u = 3t \quad dv = e^{2t}$$

$$\frac{du}{dt} = 3$$

$$v = \frac{1}{2} e^{2t}$$

$$du = 3dt$$

$$\int 3te^{2t} \, dt = 3t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \cdot 3 \, dt$$

$$\int 3te^{2t} \, dt = \frac{3t}{2} e^{2t} - 3 \int \frac{1}{2} e^{2t} \, dt$$

$$\int 3te^{2t} \, dt = \left[\frac{3t}{2} e^{2t} - \frac{3}{4} e^{2t} \right] + C$$

$$8 \int x^2 \sin x \, dx$$

$$\text{Let } u = x^2 \quad du = 2x \, dx$$

$$\frac{du}{dx} = 2x$$

$$v = -\cos x$$

$$du = 2x \, dx$$

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int -\cos x \cdot 2x \, dx$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \int \cos x \cdot x \, dx \quad \text{--- (1)}$$

$$\int \cos x \cdot x \, dx$$

$$\text{Let } u = x, \quad dv = \cos x$$

$$\frac{du}{dx} = 1$$

$$v = \sin x$$

$$du = dx$$

$$\int \cos x \cdot x \, dx = x \sin x - \int \sin x \cdot dx \quad \text{--- (2)}$$

Putting eq (2) into eq (1)

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \left[x \sin x - \int \sin x \cdot dx \right]$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \left[x \sin x - \{-\cos x\} \right] + C$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \left[x \sin x + \cos x \right] + C$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$4 \int \cos 5x \cos 6x dx$$

$$\text{Let } A = 5x \quad B = 6x$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int \cos(5x+6x) + \cos(5x-6x)$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \int \cos 11x - \cos x$$

$$\int \cos 5x \cos 6x dx = \frac{1}{2} \left[\frac{\sin 11x}{11} - \sin x \right] + C$$

$$\int \cos 5x \cos 6x dx = \frac{\sin 11x}{22} - \frac{\sin x}{2} + C$$

$$5 \int \sin 7x \cos 2x dx$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$\int \sin 7x \cos 2x dx = \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right] + C$$

$$\int \sin 7x \cos 2x dx = -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$