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# Solution

1) using simplex method

$Z = 2x + y$

constraints: $L_1: x + 2y + S_1 = 10$

$L_2: x + y + S_2 = 6$

$L_3: x - y + S_3 = 2$

$L_4: x - 2y + S_4 = 1$

in tabular form

| SV | Product | | SV | | | | Solution |
|---|---|---|---|---|---|---|---|
| | $x$ | $y$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | quantity |
| $L_1$ | 1 | 2 | 1 | 0 | 0 | 0 | 10 |
| $L_2$ | 1 | 1 | 0 | 1 | 0 | 0 | 6 |
| $L_3$ | 1 | -1 | 0 | 0 | 1 | 0 | 2 |
| $L_4$ | 1 | -2 | 0 | 0 | 0 | 1 2 | 1 2 |
| $Z$ | 2 | 1 | 0 | 0 | 0 | 0 | |

$x$ is the pivot column and 2, $L_4$ is the pivot element.

| SV | Product | | SV | | | | Solution | |
|---|---|---|---|---|---|---|---|---|
| | $x$ | $y$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | quantity | |
| $L_1$ | 0 | 4 | 1 | 0 | 0 | -2 1 | 9 | 9/4 1 1/4 |
| $L_2$ | 0 | -1 | 0 | 1 | 0 | -2 1 | 5 | 5 |
| $L_3$ | 0 | 1 | 0 | 0 | 1 | -2 1 | 1 | 1 |
| $L_4$ | 1 | -2 | 0 | 0 | 0 | 1 | 1 | |
| $Z$ | 0 | 5 | 0 | 0 | 0 | -2 | -2 | |

Y is the new pivot column and -3 makes $L_2$ the new pivot element

| SV | Product | | SV | | | | Solution Quantity |
|---|---|---|---|---|---|---|---|
| | x | y | $S_1$ | $S_2$ | $S_3$ | $S_4$ | |
| $L_1$ | 0 | 0 | 1 | 4 | 0 | -5 | 29 |
| $L_2$ | 0 | 1 | 0 | -1 | 0 | 1 | -5 |
| $L_3$ | 0 | 0 | 0 | 1 | 1 | -2 | 6 |
| $L_4$ | 1 | 0 | 0 | -2 | 0 | -1 | 11 |
| Z | 0 | 0 | 0 | 5 | 0 | -7 | -23 |

answer:

| SV | Product | | SV | | | | Solution Quantity |
|---|---|---|---|---|---|---|---|
| | x | y | S1 | S2 | S3 | S4 | |
| $L_1$ | 0 | 0 | 1 | 4 | 0 | -5 | 0 |
| $L_2$ | 0 | 1 | 0 | -1 | 0 | 1 | -5 |
| $L_3$ | 0 | 0 | 0 | 1 | 1 | -2 | 6 |
| $L_4$ | 1 | 0 | 0 | -2 | 0 | -1 | 11 |
| Z | 0 | 0 | 0 | 5 | 0 | -7 | -23 |

2)

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | SS |
|---|---|---|---|---|---|
| $S_1$ | 20 | 30 | 110 | 70 | 60 |
| $S_2$ | 10 | 0 | 60 | 10 | 10 |
| $S_3$ | 50 | 80 | 150 | 90 | 100 |
| DD | 70 | 50 | 30 | 20 | |

Is the demand equal to supply?

(70 + 50 + 30 + 20) = (60 + 10 + 100)

170 = 170

1) the regret method ALSO known as vogel method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply | Row $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | 20 (10) | 30 (50) | 110 | 70 | 60 10 | 10 | 10 | 50 |
| $S_2$ | 10 | 9 | 60 | (10) 10 | 10 | 10 | — | |
| $S_3$ | 50 (60) | 80 | (30) 150 | (10) 90 | ~~100~~ 40 0 | 30 | 30 | 40 |
| Demand | 70 0 | 50 0 | 30 0 | 20 10 0 | | | | |
| column $P_1$ | 10 | 30 | 50 | 60 | | | | |
| $P_2$ | 30 | 50 | 40 | 20 | | | | |
| $P_3$ | ~~30~~ 20 | | 40 | 20 | | | | |

$C = 20(10) + 50(60) + 30(50) + 150(30) + 10(10) + 90(10) = 200 + 3000 + 1500 + 4500 + 100 + 900 = 10200$ units

Check for degenerate problem.

occupied cells = 6

$(m+n-1) = (3+4-1) = 7-1 = 6$

2) the least cost method

| | D1 | D2 | D3 | D4 | Supply |
|---|---|---|---|---|---|
| S1 | $\boxed{60}$ 20 | 30 | 110 | 70 | 0 ~~60~~ |
| S2 | 10 | $\boxed{10}$ 0 | 60 | 10 | 0 ~~10~~ |
| S3 | $\boxed{10}$ 50 | $\boxed{40}$ 80 | $\boxed{30}$ 150 | $\boxed{20}$ 90 | 0 80 30 ~~90~~ ~~100~~ |
| Demand | 0 10 ~~70~~ | 0 ~~40~~ ~~80~~ | 30 | 0 20 | |

$C = 20(60) + 0(10) + 50(10) + 80(40) + 150(30) + 90(20)$

$= 1200 + 0 + 500 + 3200 + 4500 + 1800 = \cancel{10,120}$

$= 11,200$ ~~birr~~

Check for degenerate problem

occupied cells $= 6$

$(m+n-1) = (3+4-1) = 7-1 = 6$

3) North west corner method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | $\boxed{60}$ 20 | 30 | 110 | 70 | ~~60~~ |
| $S_2$ | $\boxed{10}$ 10 | 0 | 60 | 10 | ~~10~~ |
| $S_3$ | 50 | $\boxed{50}$ 80 | $\boxed{30}$ 150 | $\boxed{20}$ 90 | ~~100~~ 50 20 0 |
| Demand | 10 ~~70~~ | 50 | 30 | 20 | |

$C = 20(60) + 10(10) + 80(50) + 150(30) + 90(20)$
$= 1200 + 100 + 4000 + 4500 + 1800$
$= 11,600$ units

Check for degenerate problem
number of occupied cells = 6
$(m+n-1) = (3+4-1) = 7-1 = 6$

3) $A_1$ — outstanding result

$A_2$ — fair result

$A_3$ — poor result

$A_1 \longrightarrow A_1 : 0.7 = A_{11}$

$A_1 \longrightarrow A_2 : 0.3 = A_{12}$

$A_1 \longrightarrow A_3 : 0.0 = (1-0.7-0.3) = A_{13}$

$A_2 \longrightarrow A_1 : 0.2 = A_{21}$

$A_2 \longrightarrow A_2 : 0.6 = A_{22}$

$A_2 \longrightarrow A_3 : 0.2 = A_{23}$

$A_3 \longrightarrow A_1 : 0.0 = (1-0.6-0.4) = A_{31}$

$A_3 \longrightarrow A_2 : 0.4 = A_{32}$

$A_3 \longrightarrow A_3 : 0.6 = A_{33}$

9)

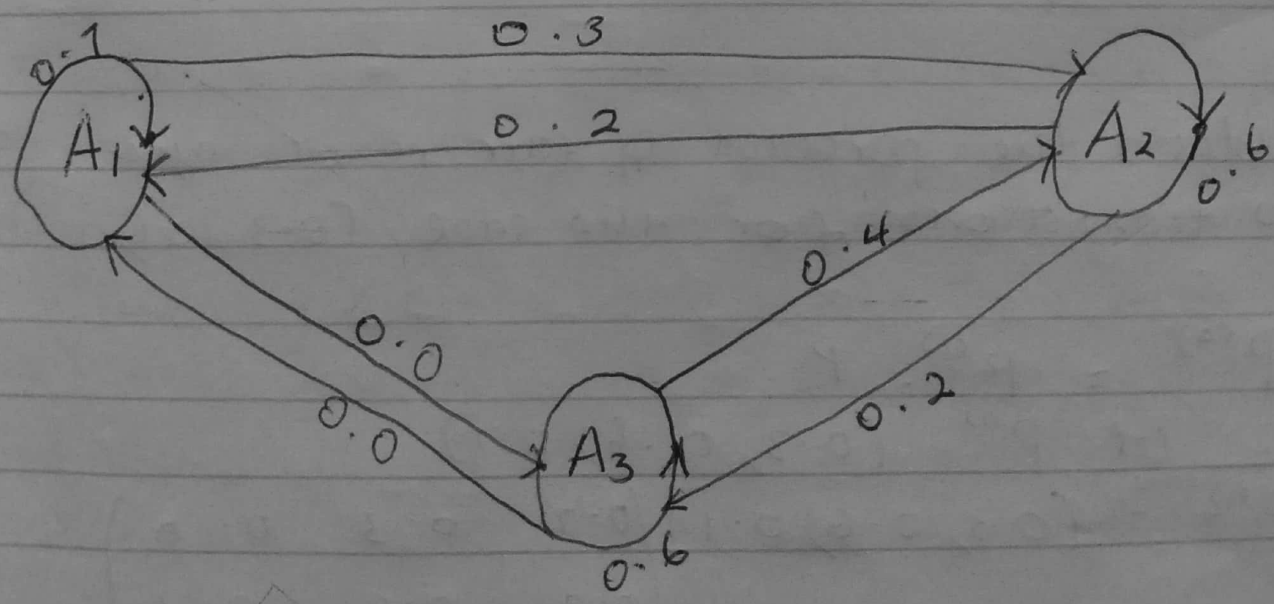


FIG: STATE TRANSITION DIAGRAM

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

FIG: TRANSITION STATE MATRIX

~~FIG:~~

Check if it is a transitional matrix

- inverse: $\begin{pmatrix} 0.7 & 0.2 & 0.0 \\ 0.3 & 0.6 & 0.4 \\ 0.0 & 0.2 & 0.6 \end{pmatrix} \neq \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$

summation of each of rows is equal to 1

$A_1$: $0.7 + 0.3 + 0.0 = 1$

$A_2$: $0.2 + 0.6 + 0.2 = 1$

$A_3$: $0.0 + 0.4 + 0.6 = 1$

b) obtain the forecast of fair results after two transitions for the case $(0.3, 0.6, 0.1)$

$$P^{(2)} = P^{(1)} \cdot P$$

let $P^{(1)} = (0.3, 0.6, 0.1)$

$$P^{(2)} = (0.3, 0.6, 0.1) \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

$$p^{(2)} = \left((0.3\times0.7)+(0.6\times0.2)+0,\ (0.3\times0.3)+(0.6\times0.6)+(0.1\times0.4),\ 0+(0.6\times0.2)+0.1\times0.6\right)$$

$$= (0.21+0.12+0,\ 0.09+0.36+0.04,\ 0+0.12+0.06)$$

$$= (0.33,\ 0.49,\ 0.18)$$

3) $p^{(3)} = p^{(3)} \cdot p$

$$(x, y, z) = (x, y, z) \cdot \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.4 \end{pmatrix}$$

$\cancel{(x, y, z) = (0.7}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.7x + 0.2y \\ 0.3x + 0.6y + 0.4z \\ 0.2y + 0.4z \end{pmatrix} \quad \begin{matrix} -(1) \\ -(11) \\ -(111) \end{matrix}$$

from (1)

$x = 0.7x + 0.2y \Rightarrow x - 0.7x = 0.2y$

$\Rightarrow 0.3x = 0.2y \quad \therefore \Rightarrow x = \frac{2}{3}y$

from (111)

$z = 0.2y + 0.4z \Rightarrow z - 0.4z = 0.2y$

$\Rightarrow 0.6z = 0.2y \Rightarrow z = \frac{2}{6}y = \frac{1}{3}y$

but $\Sigma p = 1$ ? $p^{(3)} = x, y, z$

$x + y + z = 1 \Rightarrow (\frac{2}{3}y + y + \frac{1}{3}y = 1) \times 3$

$2y + 3y + y = 3$

$6y = 3 \Rightarrow y = 3/6 = \frac{1}{2}$

$x = \frac{2}{3}(y) = \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3}$

$z = \frac{1}{3}(y) = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

$\therefore (x, y, z) = \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right)$

4) arrival rate ($\lambda$) = 200 customers in 40 hours

= 5 customers/hr

Service rate ($\mu$) = 480 customers in 80 hours

= 6 customers/hr

a) the traffic intensity $p = \frac{\lambda}{\mu} = \frac{5}{6}$ or 83.33%

b) average number of items in the queue

$E_L = \frac{p^2}{(1-p)} = \left(\frac{5}{6}\right)^2 \div \left(1 - \frac{5}{6}\right)$

$= \frac{25}{36} \div \frac{1}{6} = \frac{25}{\cancel{36}6} \times \frac{\cancel{6}}{1} = 4\frac{1}{6}$ or 4.167 items

c) average number of items in the system ~~a queue before service is rendered~~

$= E(n) = \frac{P}{1-P} = \frac{5}{6} \div \left(1 - \frac{5}{6}\right)$

$= \frac{5}{6} \div \frac{1}{6} = \frac{5}{\cancel{6}_1} \times \frac{\cancel{6}}{1} = 5$ items

d) Average time in a queue before service is rendered = average waiting time

$= \frac{1}{1-P} \quad 1 \div \left(1 - \frac{5}{6}\right) = 1 \div \left(\frac{1}{6}\right) = 1 \times 6$ hrs

$= 6$ hrs

e) average time in system $= \frac{1}{M(1-P)}$

$= 1 \div 6\left(1 \div \frac{5}{6}\right) = 1 \div \cancel{6}\left(\frac{1}{\cancel{6}}\right) = 1$ hr

5)

| Bin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | – | 4 | – | 6 | 7 | – | 3 | – | 5 |
| 2 | 4 | – | 5 | 2 | – | 3 | 1 | – | – |
| 3 | – | 5 | – | 7 | – | 2 | 2 | 4 | – |
| 4 | 6 | 2 | 7 | – | 4 | 1 | – | 3 | – |
| 5 | – | – | – | 4 | – | 1 | – | – | – |
| 6 | – | 3 | 2 | 1 | 1 | – | 2 | 2 | 4 |
| 7 | 3 | 1 | 2 | – | – | 2 | – | 5 | 2 |
| 8 | – | – | 4 | 3 | – | 2 | 5 | – | 6 |
| 9 | 5 | – | – | – | – | 4 | 2 | 6 | – |

The cells with the value of '—' are ineligible therefore I can either ignore them or set them to an unreasonably high value so they still wouldn't count in the calculation

∴ I shall replace it with the value of 1,000.

| Bin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1000 | 4 | 1000 | 6 | 7 | 1000 | 3 | 1000 | 5 |
| 2 | 4 | 1000 | 5 | 2 | 1000 | 3 | 1 | 1000 | 1000 |
| 3 | 1000 | 5 | 1000 | 7 | 1000 | 2 | 2 | 4 | 1000 |
| 4 | 6 | 2 | 7 | 1000 | 4 | 1 | ~~1000~~ 1000 | 3 | 1000 |
| 5 | 1000 | 1000 | 1000 | 4 | 1000 | 1 | 1000 | 1000 | 1000 |
| 6 | 1000 | 3 | 2 | 1 | 1 | 1000 | ~~1000~~ 2 | 2 | 4 |
| 7 | 3 | 1 | 2 | 1000 | 1000 | 2 | 1000 | 5 | 2 |
| 8 | 1000 | 1000 | 4 | 3 | 1000 | 2 | 5 | 1000 | 6 |
| 9 | 5 | 1000 | 1000 | 1000 | 1000 | 4 | 2 | 6 | 1000 |

Subtract the smallest from each column

| Bin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 997 | 3 | 998 | 5 | 6 | 999 | 2 | 998 | 3 |
| 2 | 1 | 999 | 3 | 1 | 999 | 2 | 0 | 998 | 998 |
| 3 | 997 | 4 | 998 | 6 | 999 | 1 | 1 | 2 | 998 |
| 4 | 3 | 1 | 5 | 999 | 3 | 0 | 999 | 1 | 998 |
| 5 | 997 | 999 | 998 | 3 | 999 | 0 | 999 | 998 | 998 |
| 6 | 997 | 2 | 0 | 0 | 0 | 999 | 1 | 0 | 2 |
| 7 | 0 | 0 | 0 | 999 | 999 | 1 | 999 | 3 | 0 |
| 8 | 997 | 999 | 2 | 2 | 999 | 1 | 4 | 998 | 4 |
| 9 | 2 | 999 | 998 | 999 | 999 | 3 | 1 | 4 | 998 |

Subtract the smallest from each row

| Bin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 995 | 1 | 996 | 3 | 4 | 997 | 0 | 996 | 1 |
| 2 | 1 | 999 | 3 | 1 | 999 | 2 | 0 | 998 | 998 |
| 3 | 996 | 3 | 997 | 5 | 998 | 0 | 0 | 1 | 997 |
| 4 | 3 | 1 | 5 | 999 | 3 | 0 | 999 | 1 | 998 |
| 5 | 997 | 999 | 998 | 3 | 999 | 0 | 999 | 998 | 998 |
| 6 | 997 | 2 | 0 | 0 | 0 | 999 | 1 | 0 | 2 |
| 7 | 0 | 0 | 0 | 999 | 999 | 1 | 999 | 3 | 0 |
| 8 | 996 | 998 | 1 | 1 | 998 | 0 | 3 | 997 | 3 |
| 9 | 1 | 998 | 997 | 998 | 998 | 2 | 0 | 3 | 997 |

Deduct 1 from the 'uncrossed' & add 1 to where the crossings meet

| Bin | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 994 | 0 | 995 | 2 | 3 | 997 | 0 | 995 | 0 |
| 2 | 0 | 998 | 2 | 0 | 998 | 2 | 0 | 997 | 997 |
| 3 | 995 | 2 | 996 | 4 | 997 | 0 | 0 | 0 | 996 |
| 4 | 2 | 0 | 4 | 998 | 2 | 0 | 999 | 0 | 999 |
| 5 | 996 | 998 | 997 | 2 | 998 | 0 | 999 | 998 | 998 |
| 6 | 997 | 2 | 0 | 0 | 0 | 1000 | 2 | 0 | 2 |
| 7 | 0 | 0 | 0 | 999 | 999 | 2 | 1000 | 3 | 0 |
| 8 | 995 | 997 | 0 | 0 | 997 | 0 | 3 | 996 | 2 |
| 9 | 0 | 997 | 996 | 997 | 997 | 2 | 0 | 2 | 996 |

(i) The Pipes can be but in the following ways

Pipe 1 : Pipe 2 = 4
Pipe 2 : Pipe 4 = 2
Pipe 3 : Pipe 7 = 2
Pipe 4 : Pipe 8 = 3
Pipe 5 : Pipe 6 = 1
Pipe 6 : Pipe 5 = 1
Pipe 7 : Pipe 9 = 2
Pipe 8 : Pipe 3 = 4
Pipe 9 : Pipe 1 = 5

$\overline{24}$

(ii) minimal cost is ₦24 million

Table I:

6)

| machine/cost | A | B | C |
|---|---|---|---|
| X | 25 | 15 | 22 |
| Y | 31 | 20 | 19 |
| Z | 35 | 24 | 17 |

(1) Show that assignment model is a special case of transportation model

The assignment problem is a special case of transportation problem where the objective is to minimize the cost or time completing a number of jobs by a number of persons, maximizes revenue & sales efficiently

Assignment becomes a problem because each job requires different skills and the capacity of each person with respect to the jobs can be different thus giving rise to cost differences.

If each person is able to do all jobs with the same efficiency then all costs will be the same and each job can be assigned to any person.

When assignment is a problem, it becomes a typical optimization problem - Therefore you can compare an assignment problem to a transportation problem. The cost element is given as a square matrix and requirement at each destination is one and availability at each origin is also one (Cook with Niksaj, 2011)

4) Table 2:

| machine/cost | A | B | C |
|---|---|---|---|
| X | 0 | 0 | 5 |
| Y | 6 | 5 | 2 |
| Z | 10 | 9 | 0 |

Table 3:

| machine/cost | A | B | C |
|---|---|---|---|
| X | 0 | 0 | 5 |
| Y | 4 | 3 | 0 |
| Z | 10 | 9 | 0 |

Subtract 3 from the "uncrossed"

Table 4:

| machine/cost | A | B | C |
|---|---|---|---|
| X | 0 | 0 | 8 |
| Y | 1 | 0 | 0 |
| Z | 7 | 6 | 0 |

~~ma~~ Assignment is:

machine Z - cost C : 17

machine Y - cost B : 20

machine X - cost A : 25

₦62

b) i) Describe Three areas of application of operation research in computing

(1) Arigculture! There are limited numbers of farmers, farmlands and resources but still, a high demand of food from the general population of a community, with the knowledge of operations research; resource allocation can be effectively performed to maximize produce & minimize costs

(11) Traffic control! Due to increasing population especially in Nigeria, traffic congestion can cause frustration and disorderliness in the country, use of proper operations research techniques to control driving queues and to plan road constructions and route mapping will make traffic control effectively done

(111) Industry: Especially in times plagued by the current pandemic of 2020, the economy and several industry will need to rebuild fallen channels and through Inventory control, assignment models, transportation models, etc. it can be done efficiently

5 (a) Describe briefly two impacts that operations research has had in any organization.

- <u>SAMSUNG</u>: applied OR and it helped to reduce manufacturing times and inventory levels and they made 200 million dollars excess

- AT&T: used OR and it helped them redesign the operations of their core centers and they made 750 million dollars excess