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MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO. 111
MAT 104 ASSIGNMENT

1. Evaluate dy/dx at $x = 2.5$, correct to 3 significant figures given $y = \frac{(2x^2+3)}{\ln 2x}$

Solution

$$\ln y = \ln(2x^2+3) - \ln(\ln 2x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{(2x^2+3)} \cdot \frac{4x}{1} - \frac{1}{\ln 2x} \cdot \frac{1}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{4x}{2x^2+3} - \frac{1}{x\ln 2x}$$

$$\frac{dy}{dx} = y\left[\frac{4x}{2x^2+3} - \frac{1}{x\ln 2x}\right]$$

$$\frac{dy}{dx} = \frac{(2x^2+3)}{\ln 2x}\left[\frac{4x}{2x^2+3} - \frac{1}{x\ln 2x}\right]$$

At $x = 2.5$,

$$\frac{dy}{dx} = \frac{2(2.5)^2+3}{\ln 2(2.5)}\left[\frac{4(2.5)}{2(2.5)^2+3} - \frac{1}{(2.5)\ln 2(2.5)}\right]$$

$$\frac{dy}{dx} = \frac{15.5}{1.6094}\left[\frac{10}{15.5} - \frac{1}{4.0235}\right]$$

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$$\frac{dy}{dx} = 9.6309(0.3966)$$

$$\therefore \ dy/dx = 3.82$$

2. Find the gradient of the curve $y = \dfrac{2x}{(x^2-5)}$ at the point $(2, -4)$

Solution

$$y = \frac{2x \longrightarrow u}{(x^2-5) \longrightarrow v}$$

$$u = 2x \ ; \ \frac{du}{dx} = 2$$

$$v = x^2 - 5 \ ; \ \frac{dv}{dx} = 2x$$

$$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x^2-5)2 - 2x(2x)}{(x^2-5)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 - 10 - 4x^2}{(x^2-5)^2}$$

$$\frac{dy}{dx} = \frac{-2x^2-10}{(x^2-5)^2} = m$$

At point, $(2,-4)$

$$m = \frac{-2(2)^2 - 10}{(2^2-5)^2}$$

$$m = \frac{-8-10}{(-1)^2}$$

$$\therefore m = -18$$

3. If $Z = 2x^3 \ln y$. Find dz/dy.

Solution

$$\ln z = \ln(2x^3) + \ln(\ln y)$$

$$\frac{1}{z} \cdot \frac{dz}{dy} = \frac{1}{2x^3} \cdot \frac{6x^2}{1} \frac{dx}{dy} + \frac{1}{\ln y} \cdot \frac{1}{y}$$

$$\frac{1}{z} \frac{dz}{dy} = \frac{6x^2}{2x^3} \frac{dx}{dy} + \frac{1}{y \ln y}$$

$$\frac{dz}{dy} = z\left[\frac{3}{x}\frac{dx}{dy} + \frac{1}{y \ln y}\right]$$

$$\frac{dz}{dy} = 2x^3 \ln y\left[\frac{3}{x}\frac{dx}{dy} + \frac{1}{y \ln y}\right]$$

4 Integrate $x(2x^2+1)^{1/2}$ with respect to x from $(0$ to $2)$

Solution

$$\int_0^2 x(2x^2+1)^{1/2}$$

$$\int_0^2 x\sqrt{2x^2+1}$$

Let,

$$u = \sqrt{2x^2+1}$$

$$u^2 = 2x^2 + 1$$

$$2x^2 = u^2 - 1$$

$$x = \sqrt{\frac{u^2-1}{2}}$$

$$x = \left(\frac{u^2-1}{2}\right)^{1/2}$$

Let $P = \dfrac{u^2-1 \rightarrow V}{2 \rightarrow W}$

$$x = P^{1/2}$$

$$dx/dP = \frac{1}{2} P^{-1/2}$$

$$\frac{dP}{du} = \frac{W\frac{dv}{du} - V\frac{dw}{du}}{W^2}$$

$V = u^2 - 1$, $\frac{dv}{du} = 2u$

$W = 2$, $dw/du = 0$

$$\frac{dP}{du} = \frac{2(2u) - (u^2-1)0}{2^2}$$

$$= u$$

Also,

$$dx/du = \frac{dP}{du} \times \frac{dx}{dP}$$

$$\frac{dx}{du} = u \times \frac{1}{2} P^{-1/2}$$

$$dx/du = \frac{u}{2} P^{-1/2}$$

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$$dx = \frac{u}{2}\left(\frac{u^2-1}{2}\right)^{-1/2} du$$

$$\Rightarrow \int_0^2 \left(\frac{u^2-1}{2}\right)^{1/2} \cdot u \cdot \frac{u}{2}\left(\frac{u^2-1}{2}\right)^{-1/2} du$$

$$= \int_0^2 \frac{u^2}{2} du$$

$$= \frac{1}{2}\int_0^2 u^2 \, du$$

$$= \frac{1}{2}\left[\frac{u^3}{3}\right]_0^2$$

$$= \frac{u^3}{6}\Bigg]_0^2 = \frac{\left([2x^2+1]^{1/2}\right)^3}{6}\Bigg]_0^2$$

$$= \frac{(2x^2+1)^{3/2}}{6}\Bigg]_0^2$$

$$= \left[\frac{\sqrt{(2(2)^2+1)^3}}{6}\right] - \left[\frac{\sqrt{(2(0)^2+1)^3}}{6}\right]$$

$$= \frac{\sqrt{729}}{6} - \frac{\sqrt{1}}{6}$$

$$= \frac{27}{6} - \frac{1}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$