Irene Obrozie

17/SCI01/057

CSC314 PRACTICE QUESTIONS;

Question 1

Irene Obrozie , 17/SCI01/057

Question 1

Find the maximum value of $z = 2x + y$

Subject to $x + 2y \leq 10$, $x - y \leq 2$

$x + y \leq 6$ $x - 2y \leq 1$

Solution

a) Obj fn : $z = 2x + y$

b) Constraints ; $x + 2y \leq 10$

$x + y \leq 6$

$x + 2y \leq 10$

when $x = 0$, $y = 5$ $(0, 5)$

$x + 2y \leq 10$

when $y = 0$, $x = 10$ $(10, 0)$

$x - 2y \leq 2$

when $x = 0$, $y = -1$ (0,

when $y = 0$, $x = 2$ (2,

$x + y \leq 6$

when $x = 0$, $y = 6$ $(0, 6)$

when $y = 0$, $x = 6$ $(6, 0)$

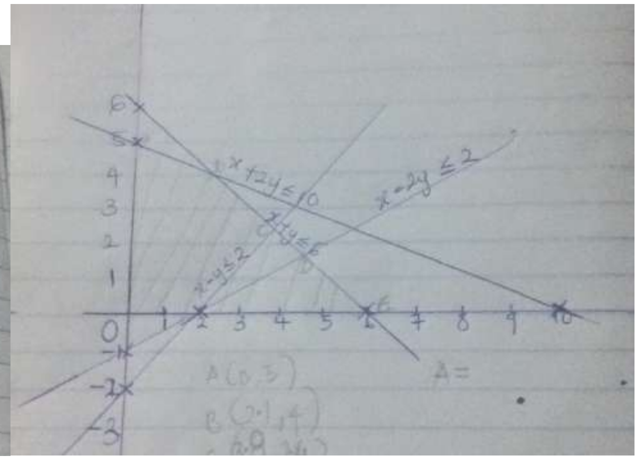
$x - y \leq 2$

when $x = 0$, $y = -2$ $(0, -2)$

when $y = 0$, $x = 2$ $(2, 0)$

A (0, 5) $z = 2(0) + 5 = 5$
B (2.1, 4) $z = 2(2.1) + 4 = 8.2$
C (3.8, 2.4) $z = 2(3.8) + 2.4 = 10$
D (4.5, 1.8) $z = 2(4.5) + 1.8 = 10.8$
E (6, 0) $z = 2(6) + 0 = 12$

The optimal soln of this maximization problem is (D = 10.8).



Question 2

i) Vogel method

	D1	D2	D3	D4	S.S	RP1	RP2	RP3	RP4
S1	120	130	110	120	60	10	10	10	–
S2	110	20 120	20 180	10 110	10	10	–	–	–
S3	10 130	30 160	150	190	100	30	30	30	40
D.D	70	50	30	20					
CP1	10	30	50	60					
CP2	10	30	50	–					
CP3	30	50	–	–					
CP4	50	–	–	–					

Check for degeneracy
Occupied cells = 6
$m + n - 1$
$3 + 4 - 1$
$= 7 - 1 = 6$
∴ there's no degeneracy

Total cost for transportation =
$S_1 \to D_2 = 20 \times 30$
$= 600$
$S_1 \to D_3 = 30 \times 110$
$= 3300$
$S_1 \to D_4 = 10 \times 70$
$= 700$
$S_2 \to D_4 = 10 \times 10$
$= 100$
$S_3 \to D_1 = 70 \times 50$
$= 3500$
$S_3 \to D_2 = 30 \times 80$
$= 2400$

$= 600 + 3300 + 700 + 100 + 3500 + 2400$
$= 10{,}600$

ii) LCM

	D1	D2	D3	D4	S.S
S1	20 100	30	110	70	600
S2	10 110	0 110	60 130	10	100
S3	50	80 40	150	90 120	100 30 50
D.D	70 20 0	50 40 0	30 0	20 0	70 0

Check for degeneracy =
Occupied cells = 6
m+n-1
3+4-1
= 7-1 = 6
∴ There's no degeneracy

Total Transportation
$S_1 \rightarrow D_1 = 20 \times 60 = 1200$
$S_2 \rightarrow D_2 = 0 \times 10 = 0$
$S_3 \rightarrow D_1 = 50 \times 10 = 500$
$S_3 \rightarrow D_2 = 80 \times 40 = 3200$
$S_3 \rightarrow D_3 = 150 \times 30 = 4500$
$S_3 \rightarrow D_4 = 90 \times 20 = 1800$

$= 1200 + 500 + 3200 + 4500 + 1800$
$= 11,200$

(iii) NWCM

	D_1	D_2	D_3	D_4	S.S
S_1	20 (60)	30	110	70	~~60~~ 0
S_2	10 (10)	0	60	10	~~10~~ 0
S_3	50	80 (50)	150 (30)	90 (20)	100 ~~50~~ 20 0
D.D	~~70~~ 10 0	~~50~~ 0	30	20	

Total Transportation
$S_1 \rightarrow D_1 = 60 \times 20 = 1200$
$S_2 \rightarrow D_1 = 10 \times 10 = 100$
$S_3 \rightarrow D_2 = 80 \times 50 = 4000$
$S_3 \rightarrow D_3 = 150 \times 30 = 4500$
$S_3 \rightarrow D_4 = 90 \times 20 = 1800$

$= 1200 + 100 + 4000 + 4500 + 1800$
$= 11,600$

Question 3

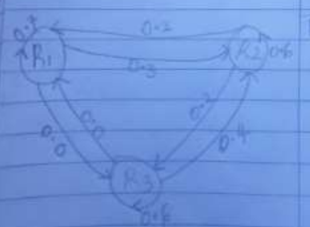
R_1 = Outstanding
R_2 = Fair
R_3 = Poor

QUESTION 3

$R_1 \to R_1 \Rightarrow P_{11} = 0.7$
$R_1 \to R_2 \Rightarrow P_{12} = 0.3$
$R_2 \to R_1 \Rightarrow P_{21} = 0.2$
$R_2 \to R_2 \Rightarrow P_{22} = 0.6$
$R_2 \to R_3 \Rightarrow P_{23} = 0.2$
$R_3 \to R_2 \Rightarrow P_{32} = 0.4$
$R_3 \to R_3 \Rightarrow P_{33} = 0.6$

a) Presenting the information in directed graphs and transition Matrix

- Directed graph



- Transition Matrix

$$P = \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

b)

i) The forecast of fair results after 2 transitions for the case (0.3, 0.6, 0.1)

Soln

$$p^{(1)} = p^{(0)} \cdot P$$
$$p^{(1)} = (0.3, 0.6, 0.1)\begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$
$$= (0.33, 0.49, 0.18)$$

$$p^{(2)} = p^{(1)} \cdot P$$
$$p^{(2)} = (0.33, 0.49, 0.18)\begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$
$$= (0.329, 0.463, 0.206)$$

c) Determining the steady state vector the transition matrix in (i)

$$P = \begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

Soln

$p^{(2)} = (x, y, z)$

$p^{(\infty)} = p^{(\infty)} \cdot P$

$$(x, y, z) = (x, y, z)\begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.4 & 0.6 \end{pmatrix}$$

$(x, y, z) = (0.7x + 0.2y + 0z\ ,\ 0.3x + 0.6y + 0.4z,$
$0.0x + 0.2y + 0.6z)$

$x = 0.7x + 0.2y + 0z$ — ①
$y = 0.3x + 0.6y + 0.4z$ — ②
$z = 0.0x + 0.2y + 0.6z$ — ③

$\Sigma p = 1$
$p^{(\infty)} = (x, y, z)$
$x + y + z = 1$ — ④

From ①
$0.2y = x - 0.7x$
$0.2y = 0.3x$
$x = \frac{0.2y}{0.3}$

$x = \frac{2}{3}y$

From eqn ③
$z = 0.2y + 0.6z$
$0.2y = z - 0.6z$
$0.2y = 0.4z$
$y = \frac{0.4z}{0.2}$

$y = 2z$

From eqn ②
$y = 0.3x + 0.6y + 0.4z$
$0.4y = 0.3x + 0.4z$

$x = \frac{2}{3}y$ — ①

$0.4y = 0.3x + 0.4z$ — ②
$y = 2z$ — ③

$x + y + z = 1$ — ④

Substitute $y = 2z$, $x = 2/3\,y$ into eqn 4
$\left(\frac{2}{3}y\right) + (2z) + z = 1$

$\frac{2y + 9z}{3} = 1$

$2y + 9z = 3$ — ⑤
Input $y = 2z$ into eqn ⑤

$2(2z) + 9z = 3$

$4z + 9z = 3$

$13z = 3$

$z = \frac{3}{13}$

Put $z = 3/13$ into (3)

$y = 2z$

$y = 2 \times \frac{3}{13}$

$y = \frac{6}{13}$

Put $y = 6/13$ into (1)

$x = \frac{2}{3} y$

$x = \frac{2}{\cancel{3}} \times \frac{\cancel{6}\,2}{13}$

$x = \frac{4}{13}$

$\therefore x = 4/13,\ y = 6/13,\ z = 3/13$

Question 4

QUESTION 4

1) Arrival rate $= \lambda = \frac{200}{40} = 5$

Service rate $= \mu = \frac{480}{80} = 6$

a) Traffic intensity $= \frac{\lambda}{\mu} = \frac{5}{6} = 0.83$

b) Avg no of items in the queue $= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{5^2}{6(6-5)}$

$= 4.17$

c) Avg number of items in the system $= \frac{\lambda}{\mu-\lambda} = \frac{5}{6-5}$

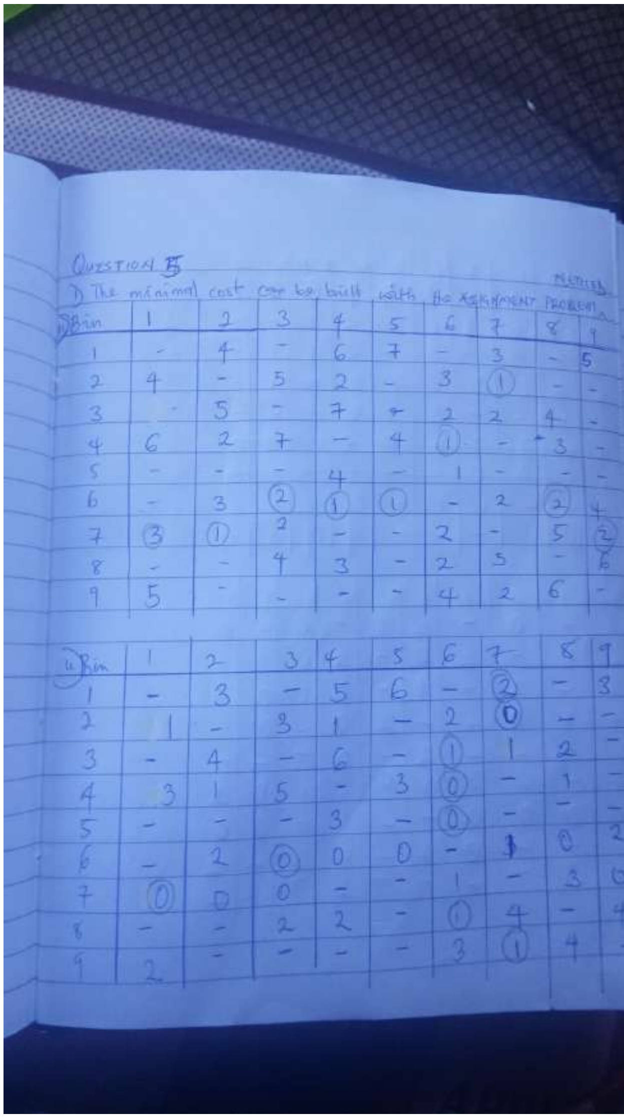
$= 5$

d) Avg time in the queue before service is rendered $= \frac{\lambda}{\mu(\mu-\lambda)}$

$= \frac{5}{6(6-5)} = 0.83$

e) Avg time in a system $= \frac{1}{(\mu-\lambda)} = \frac{1}{(6-5)} = 1$

Question 5



QUESTION 5

i) The minimal cost can be built with the ASSIGNMENT PROBLEM METHOD

ii) Bin	1	2	3	4	5	6	7	8	9
1	-	4	-	6	7	-	3	-	5
2	4	-	5	2	-	3	1	-	-
3	-	5	-	7	-	2	2	4	-
4	6	2	7	-	4	1	-	3	-
5	-	-	-	4	-	1	-	-	-
6	-	3	2	1	1	-	2	2	4
7	3	1	2	-	-	2	-	5	2
8	-	-	4	3	-	2	5	-	6
9	5	-	-	-	-	4	2	6	-

iii) Bin	1	2	3	4	5	6	7	8	9
1	-	3	-	5	6	-	2	-	3
2	1	-	3	1	-	2	0	-	-
3	-	4	-	6	-	1	1	2	-
4	3	1	5	-	3	0	-	1	-
5	-	-	-	3	-	0	-	-	-
6	-	2	0	0	0	-	1	0	2
7	0	0	0	-	-	1	-	3	0
8	-	-	2	2	-	1	4	-	4
9	2	-	-	-	-	3	1	4	-

Bin	1	2	3	4	5	6	7	8	9
1	-	1	-	3	4	-	0	-	1
2	1	-	3	1	-	2	0	-	-
3	-	3	-	5	-	0	0	1	-
4	3	1	5	-	3	0	-	1	-
5	-	-	-	3	-	0	-	-	-
6	-	2	0	0	0	-	1	0	2
7	0	0	0	-	-	1	-	3	0
8	-	-	1	1	-	0	3	-	3
9	1	-	-	-	-	2	0	3	-

Bin	1	2	3	4	5	6	7	8	9
1	-	0	-	2	3	-	0	-	0
2	0	-	2	0	-	2	0	-	-
3	-	2	-	4	-	0	0	0	-
4	2	0	4	-	2	0	-	0	-
5	-	-	-	2	-	0	-	-	-
6	-	2	0	0	0	-	1	0	2
7	0	0	0	-	-	1	-	3	0
8	-	-	0	0	-	0	3	-	2
9	0	-	-	-	-	2	0	2	-

Now we carry out the Assignment as follows;

9 : 7 = 1

8 : 4 = 1

7 : 3 = 2

6 : 5 = 1

5 : 6 = 1

4 : 2 = 1

3 : 8 = 2

2 : 1 = 3

1 : 9 = 2

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Question 6

QUESTION 6

a)

	X	B	C
X	25	15	22
Y	31	20	19
Z	35	24	17

Circling and subtracting the smallest no on each column from the others on the ...

	X	B	C
X	0	0	5
Y	6	5	2
Z	10	9	0

Circling and subtracting the smallest no on each row from others on the row

	X	B	C
X	0	0	5
Y	4	3	0
Z	10	9	0

- Crossing out the zeros in each row/column.
- Circle smallest uncrossed no. Add it to the meeting point (3) and subtract from the uncrossed nos.

	A	B	C
X	0	0	8
Y	1	0	0
Z	7	6	0

ii) X : C = 17

Y : B = 15

Z : A = 25

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6.) Areas of impact of OR
i) Transportation inventory planning.
ii) Computer Operations
iii) Communication Operations.

Impact of OR

1) Fedex used OR or applied OR to the logical planning of their shipment.

2) Samsung electronics applied OR and it helped them to reduced manufacturing time and inventory level and they made 200 million dollars.