

Ogenyi Emmanuel Ori

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ELECTRICAL ELECTRONICS ENGINEERING

MAT 104

1.)  $y = \sin\left(\frac{6}{x^2}\right)$

let  $u = 6/x^2 \dots \dots \dots (1)$

$y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$\frac{\Delta y}{\Delta u} = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}$

$\frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$\lim_{\Delta u \rightarrow 0} \left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \rightarrow 0} \left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \lim_{\Delta u \rightarrow 0} \left[\frac{\sin(\Delta u/2)}{\Delta u/2}\right]$

$\frac{dy}{du} = \cos u$

from equation (1)  $u = 6/x^2$

$u + \Delta u = \frac{6}{(x + \Delta x)^2}$

$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$

$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$

$\Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$

$$\frac{\Delta y}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \rightarrow 0} \left( \frac{\Delta y}{\Delta x} \right) = \lim_{\Delta x \rightarrow 0} \left[ \frac{-12x - 6(\Delta x)}{x^4} \right]$$

$$= \frac{dy}{dx} = \frac{-12}{x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \cdot \frac{x-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12 \cos u}{x^3}$$

For putting the value of  $u$  back

$$\frac{dy}{dx} = \frac{-12 \cos(6/x^2)}{x^3}$$

2.)  $x = 4t^3 - t^2$ ,  $y = t^4 + 2t^2$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dy}{dt} = (12t^2 - 2t) dt$$

Let  $A$  represent the area

$$A = \int y dx$$

$$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t) dt$$

$$A = \int_1^3 (12t^6 - 2t^3 + 24t^4 - 4t^2) dt$$

$$\Rightarrow \left[ \frac{12t^7}{7} - \frac{t^4}{2} + 24t^5 - 4t^3 \right]_1^3 + C$$

$$\left( \frac{26344}{7} - \frac{243}{2} + 5832 - 81 \right) - \left( \frac{12}{7} - \frac{1}{2} + \frac{24}{5} - 1 \right)$$

$$= \frac{160704}{35} - 544$$

$$A = 4586.36 \text{ sq units}$$

3.)  $x = 4t^3 - t^2$ ,  $y = t^4 + 2t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 - 4t$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$$

$$\frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$