3.) $Z = 2x^3 \ln y$

$\ln z = \ln(2x^3) + \ln(\ln y)$

$$\frac{1}{z} \cdot \frac{dz}{dy} = \frac{1}{2x^3} \cdot (6x^2) + \frac{1}{\ln y} \cdot \frac{1}{y}$$

$$\frac{1}{z}\frac{dz}{dy} = \frac{6x^2}{2x^3} + \frac{1}{y(\ln y)}$$

$$\frac{dz}{dy} = z\left[\frac{3}{x} + \frac{1}{y(\ln y)}\right]$$

$$\frac{dz}{dy} = 2x^3 \ln y \left[\frac{3}{x} + \frac{1}{y(\ln y)}\right]$$

4.) $\int_0^2 x(2x^2+1)^{1/2}\,dx$

$u = 2x^2 + 1$

$\frac{du}{dx} = 4x$, $dx = \frac{du}{4x}$

$\therefore \int_0^2 x(2x^2+1)^{1/2}\,dx$

$$\Rightarrow \int_0^2 x u^{1/2} \cdot \frac{du}{4x} = \frac{1}{4}\int_0^2 u^{1/2} \cdot du$$

$$= \frac{1}{4}\left[\frac{u^{3/2}}{\frac{3}{2}}\right] = \Big|_0^2 \, 3/8\, u^{3/2} + c$$

$$\Rightarrow \Big|_0^2 \, 3/8\,(2x^2+1)^{3/2} + c$$

$$= \frac{3(9)^{3/2}}{8} - \frac{3(1)^{3/2}}{8} + c$$

$$= \frac{81}{8} - \frac{3}{8} = \frac{78}{8}$$

i) $y = \frac{(2x^2+3)}{\ln 2x}$

$$\ln y = \ln(2x^2+3) - \ln(\ln 2x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2x^2+3} \cdot 4x - \frac{1}{\ln 2x} \cdot \frac{\cancel{2}}{\cancel{2}x}$$

$$\frac{dy}{dx} = y\left(\frac{4x}{2x^2+3} - \frac{1}{x \ln 2x}\right)$$

At $x = 2.5$

$$\frac{dy}{dx} = \frac{2(2.5)^2+3}{\ln 2(2.5)}\left(\frac{4(2.5)}{2(2.5)^2+3} - \frac{1}{2.5\ln(2 \times 2.5)}\right)$$

$$\frac{dy}{dx} \Rightarrow 3.82 \text{ to } 3 \text{ s.f}$$

2.) Gradient of $y = \frac{2x}{(x^2-5)}$ at $(2, -4)$

Let $u = 2x$, $v = x^2 - 5$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x^2-5)2 - 2x(2x)}{(x^2-5)^2}$$

$$\Rightarrow \frac{2x^2 - 10 - 4x^2}{(x^2-5)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-2(x^2+5)}{(x^2-5)^2}$$

At $(2, -4)$

$\frac{dy}{dx} = -18$