NAME: ALONYENU TREASURE AGBENU-OWO

DEPARTMENT: CIVIL ENGINEERING

MARTIC NUMBER: 19/ENG03/005

MAT 104 ASSIGNMENT

✓ Abonyenne Treasure Agbenu-Oun
✓ Civil Engineering
✓ 19/ENG03/005
✓ MAT 104 Assignment

① $y = \sin(6/x^2)$

let $u = 6/x^2$ .... ①

$\therefore y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - \cancel{\sin u}\ y$

$\Delta y = \sin(u + \Delta u) - \cancel{\sin y}\ \sin u$

$$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\frac{\Delta y}{\Delta u} = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \frac{\sin\left(\frac{\Delta u}{2}\right)}{\Delta u/2} \times \frac{1}{2}$$

$$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \frac{\sin\left(\frac{\Delta u}{2}\right)}{\Delta u/2}$$

$$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \cdot \lim_{\Delta u \to 0}\left[\frac{\sin\left(\Delta u/2\right)}{\Delta u/2}\right]$$

$$\therefore \frac{dy}{du} = \cos u$$

From equation ①; $u = 6/x^2$

$$u + \Delta u = \frac{6}{(x + \Delta x)^2}$$

$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$\therefore \Delta u = \dfrac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$

$\dfrac{\Delta u}{\Delta x} = \dfrac{-12x - 6(\Delta x)}{x^4}$

$\lim_{\Delta x \to 0} \left(\dfrac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0} \left[\dfrac{-12x - 6(\Delta x)}{x^4}\right]$

$\therefore \dfrac{du}{dx} = \dfrac{-12}{x^3}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{dy}{dx} = \cos u \times \dfrac{-12}{x^3}$

$\dfrac{dy}{dx} = \dfrac{-12\cos u}{x^3}$

Putting the value of u back

$\therefore \dfrac{dy}{dx} = \dfrac{-12\cos(6/x^2)}{x^3}$

(2) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\dfrac{dx}{dt} = 12t^2 - 2t$, $dx = (12t^2 - 2t)dt$

let A represent the area

$A = \int y\,dx$

$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$

$\therefore A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$\Rightarrow \Big|_1^3 \dfrac{12t^7}{7} - \dfrac{t^6}{3} + \dfrac{24t^5}{5} - t^4 + C$

$$\left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81\right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right)$$

$$= \frac{160704}{35} - \frac{544}{105}$$

$$\therefore A = 4586.36 \text{ sq units}$$

(3) $x = 4t^3 - t^2, \quad y = t^4 + 2t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 - 4t$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t}$$

$$= \frac{4t(t^2 - 1)}{2t(6t - 1)}$$

$$\therefore \frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$