

Maths 104

Mechanical Engineering

$$(1) y = \frac{(2x^2 + 3)}{\ln 2x}$$

$$u = 2x^2 + 3$$

$$v = \ln 2x$$

$$\frac{du}{dx} = 4x$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{4x \ln 2x - \frac{1}{x}(2x^2 + 3)}{(\ln 2x)^2}$$

$$\frac{dy}{dx} \Big|_{x=2.5} = \frac{4(2.5) \ln 2(2.5) - \frac{1}{2.5} \{2(2.5)^2 + 3\}}{[\ln 2(2.5)]^2}$$

$$\frac{dy}{dx} \Big|_{x=2.5} = \frac{16 \cdot 0.94 - 6 \cdot 2}{2.5903}$$

$$= 3.8196$$

$$\approx 3.82 \text{ (3.S.F.)} \quad \checkmark$$

$$(2) y = \frac{2x}{x^2 - 5}$$

$$u = 2x$$

$$v = x^2 - 5$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 2x$$

$$(2) \frac{dy}{dx} = \frac{2(x^2 - 5) - (2x \cdot 2x)}{(x^2 - 5)^2}$$

$$= \frac{2x^2 - 10 - 4x^2}{(x^2 - 5)^2} = \frac{2x^2 - 4x^2 - 10}{(x^2 - 5)^2}$$

$$= \frac{-2x^2 - 10}{(x^2 - 5)^2}$$

$$\frac{dy}{dx} \Big|_{(2, -4)} = m = \frac{-2(2)^2 - 10}{[(2)^2 - 5]^2} = \frac{-18}{1}$$

$$\text{Gradient} = -18 \quad \checkmark$$

$$(3) \quad z = 2x^3 \ln y$$

$$u = 2x^3 \quad v = \ln y$$
$$\frac{du}{dx} = 6x^2 \quad \frac{dv}{dy} = \frac{1}{y}$$

$$\frac{dz}{dy} = 2x^3 \cdot \frac{1}{y} + \ln y \cdot 6x^2 \frac{dx}{dy}$$

$$\therefore \frac{dz}{dy} = \frac{2x^3}{y} + 6x^2 \ln y \frac{dx}{dy}$$

$$(4) \quad \int_0^2 x(2x^2+1)^{1/2} dx$$

$$\text{let } u = 2x^2 + 1$$

$$\frac{du}{dx} = 4x$$

$$du = 4x dx$$

$$dx = \frac{du}{4x}$$

$$\int_0^2 x \cdot u^{1/2} \cdot \frac{du}{4x}$$

$$\frac{1}{4} \int_0^2 u^{1/2} du$$

$$\frac{1}{4} \left[\frac{u^{3/2}}{3/2} \right]_0^2 = \frac{1}{6} \left[u^{3/2} \right]_0^2$$

$$= \frac{1}{6} \left[(2x^2+1)^{3/2} \right]_0^2$$

$$= \frac{1}{6} \left[(2(2^2)+1)^{3/2} - (2(0)+1)^{3/2} \right]$$

$$= \frac{1}{6} [27 - 1]$$

$$= \frac{1}{6} [26]$$

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$$= 4.3333 \approx 4.33$$