MATRIC NUMBER: 17/MHS01/314
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LEVEL: 200 Level
DEPARTMENT: Mechanical Engineering
COURSE: MAT 104

1. Given $y = \frac{(2x^2+3)}{\ln 2x}$

Find dy/dx at $x = 2.5$ (3 Sig. fig.)

SOLUTION

$$y = \frac{(2x^2+3)}{\ln 2x}$$

Using quotient rule $\quad dy/dx = \frac{V\,du/dx - U\,dv/dx}{V^2}$

$U = 2x^2+3 \quad du/dx = 4x$

$V = \ln 2x \quad dv/dx = 1/x$

$$\Rightarrow dy/dx = \frac{\ln 2x\,(4x) - (2x^2+3)(1/x)}{(\ln 2x)^2}$$

$$= \frac{4x \ln 2x - 2x - 3/x}{(\ln 2x)^2}$$

$$\therefore dy/dx = \frac{4x^2 \ln 2x - 2x^2 - 3}{x(\ln 2x)^2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2.5} = \frac{4(2.5)^2 \times \ln 2(2.5) - 2(2.5)^2 - 3}{2.5(\ln 2(2.5))^2}$$

$$= \frac{30.736}{6.476}$$

$$= 4.746$$

$$\simeq 4.75 \text{ (3sf.)}$$

2. Given $y = \frac{2x}{(x^2-5)}$

Find the gradient at $(2, -4)$

SOLUTION

~~Using quotient~~

Using quotient rule,

$u = 2x \quad du/dx = 2$

$v = x^2 - 5 \quad dv/dx = 2x$

$\Rightarrow dy/dx = \frac{(x^2-5)2 - 2x(2x)}{(x^2-5)^2}$

$= \frac{2x^2 - 10 - 4x^2}{(x^2-5)^2}$

$= \frac{-2x^2 - 10}{(x^2-5)^2}$

$\therefore dy/dx = \frac{-2(x^2+5)}{(x^2-5)^2}$

$\left.\frac{dy}{dx}\right|_{x=2;\, y=-4} = \frac{-2[(2)^2+5]}{[(2)^2-5]^2}$

$= \frac{-18}{1}$

$= -18$

3. Given $z = 2x^3 \ln y$. Find dz/dy

SOLUTION

$$\frac{dz}{dy} = 2x^3\left(\frac{1}{y}\right)$$

$$= \frac{2x^3}{y}$$

4. Given $x(2x^2+1)^{1/2}$, integrate with respect to x (0 to 2)

SOLUTION

$$\int_0^2 x(2x^2+1)^{1/2} \cdot dx$$

let $u = 2x^2 + 1$

$du/dx = 4x$

$du = 4x \cdot dx$

$dx = 4x \cdot du$

$$\Rightarrow \int_0^2 x(2x^2+1)^{1/2} \cdot dx \quad \cancel{\int_0^{2x} \frac{du}{4x} \cdot u^{1/2} \cdot du \cdot 4x}$$

$$= \int_0^2 \left(\frac{1}{4} \cdot d(2x^2+1)\right)(2x^2+1)^{1/2} \cdot dx$$

$$= \frac{1}{4}\int_0^2 d(2x^2+1)\,(2x^2+1)^{1/2} \cdot dx$$

$$= \frac{1}{4}\left[\frac{(2x^2+1)^{3/2}}{3/2}\right]_0^2$$

$$= \left[\frac{(2x^2+1)^{3/2}}{6}\right]_0^2$$

$$= \left[\frac{[2(4)+1]^{3/2}}{6}\right] - \left[\frac{[2(0)+1]^{3/2}}{6}\right]$$

$$= \frac{27}{6} - \frac{1}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$