

ENG 214 (Fluid mechanics)

Assignment

1) $z_1 = 0$

$z_2 = 2.0\text{m}$

$V_1 = 5\text{m/s}$

$V_2 = 2\text{m/s}$

Pressure, $\frac{P_1}{\rho} = 2.5\text{m}$

$\frac{P_2}{\rho} = ?$

$$h_e = \frac{0.35(V_1 - V_2)^2}{2g}$$

$$= \frac{0.35(5 - 2)^2}{2 \times 9.81} = 0.1606\text{m}$$

Applying Bernoulli's Equation

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 + h_e$$

$$\frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 - \frac{V_2^2}{2g} - z_2 - h_e$$

$$= 2.5 + \frac{5^2}{2 \times 9.81} + 0 - \frac{2^2}{2 \times 9.81} - 2 - 0.1606$$

$$\frac{P_2}{\rho} = 2.5 + 1.274 - 0.204 - 0.1606 - 2.0$$

$$= \underline{1.4094\text{m}} \quad \underline{\underline{1.41\text{m}}}$$

$$2) \quad d_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$d_2 = 10 \text{ cm} = 0.1 \text{ m}$$

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (0.2)^2}{4} = 0.0314 \text{ m}^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785 \text{ m}^2$$

$$P_1 = 17.658 \text{ N/cm}^2 = 176580 \text{ N/m}^2$$

$$P_2 = 30 \text{ cm of mercury} = 0.3 \text{ m of Hg}$$

$$0.3 \text{ m Hg} \times 13.6 = 4.08$$

$$h_2 = 4.08 \text{ m}$$

$$h_1 = \frac{P_1}{\rho g} = \frac{176580}{1000 \times 9.81} = 18 \text{ m}$$

$$h = h_1 - h_2$$

$$h = 18 - (-4.08) = 22.08$$

$$\text{using } Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$Q = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 22.08}$$

$$Q = \frac{0.00503}{0.0304}$$

$$Q = 0.1654 \text{ m}^3/\text{s}$$

3) Orifice diameter; $d_o = 15\text{cm} = 0.15\text{m}$

Pipe diameter $d_i = 80\text{cm} = 0.3\text{m}$ $C_d = 0.64$

$$A_o = \frac{\pi d_o^2}{4} = \frac{\pi (0.15)^2}{4} = 0.01767\text{m}^2$$

$$A_i = \frac{\pi d_i^2}{4} = \frac{\pi (0.3)^2}{4} = 0.0706\text{m}^2$$

Differential head; $y = 50\text{cm} \Rightarrow 0.5\text{m}$

$$h = \frac{p}{\omega} = y \left(\frac{\text{s.g. of mercury}}{\text{s.g. of oil}} - 1 \right)$$

$$= 0.5 \left(\frac{13.6}{0.9} - 1 \right)$$

$$= 0.5 (14.11)$$

$$= 7.06\text{m}$$

$$\therefore h = 7.06\text{m}$$

$$\therefore Q = \frac{C_d \times A_i \times A_o}{\sqrt{A_i^2 - A_o^2}} \times \sqrt{2gh}$$

$$Q = \frac{0.64 \times 0.01767 \times 0.0706}{\sqrt{(0.0706)^2 - (0.01767)^2}} \times \sqrt{2 \times 9.8 \times 7.06}$$

$$= \frac{0.009407}{0.06844}$$

$$= 0.137\text{m}^3/\text{s}$$

4) Differential head

$$y = 170\text{mm}$$

$$y = 0.17\text{m}$$

$$\text{s.g. of mercury} = 13.6$$

$$\text{s.g. of sea water} = 1.026$$

$$\therefore h = y \left(\frac{\text{s.g. of mercury}}{\text{s.g. of sea water}} - 1 \right)$$

$$= 0.17 \left(\frac{13.6}{1.026} - 1 \right) = 2.088$$

$$\text{Velocity} = \sqrt{2g \Delta h}$$

$$V = \sqrt{2 \times 9.8 \times 2.088}$$

$$V = 6.39 \text{ m/s}$$

5. i) Volumetric efficiency = $\frac{\text{Actual flowrate}}{\text{Ideal flowrate}} \times 100$

$$\text{Actual flowrate} = \frac{0.05 \text{ m}^3/\text{min}}{60} = 0.000833 \text{ m}^3/\text{sec}$$

$$\text{Ideal flowrate} = \text{Nominal displacement} \times \text{speed}$$

$$\text{Nominal displacement} = \frac{10 \text{ cm}^3/\text{rev}}{1,000,000} = 0.00001 \text{ m}^3/\text{rev}$$

$$\text{speed} = \frac{1700 \text{ rev}/\text{min}}{60} = 28.33 \text{ rev}/\text{sec}$$

$$\therefore \text{Ideal flowrate} = 0.00001 \text{ m}^3/\text{rev} \times 28.33 \text{ rev}/\text{sec}$$

$$= 0.0002833 \text{ m}^3/\text{sec}$$

$$\text{Volumetric efficiency} = \frac{0.000833}{0.0002833} \times 100$$

$$= 294.03\%$$

ii) Fluid power; $P_F = Q \cdot dp$

$$dp = 15 \text{ bar} \times 10^5 = 1.5 \times 10^6 \text{ N/m}^2$$

$$P_F = 0.000833 \times 1.5 \times 10^6$$

$$P_F = 1,249.5 \text{ watts}$$

$$\text{iii) shaft power} = \text{Torque} \times \text{angular speed}$$

$$\text{Torque} = 15 \text{ Nm}$$

$$\text{angular speed} = 2\pi N = \frac{2 \times 22 \times 28.33}{7}$$

$$= 178.07 \text{ rad/sec}$$

$$\therefore \text{shaft power} = 15 \times 178.07$$

$$= 2,671.11 \text{ Watts}$$

$$\text{iv) Overall efficiency} = \frac{\text{Fluid power}}{\text{shaft power}} \times 100$$

$$= \frac{1,249.5}{2,671.11} \times 100$$

$$= 46.77\%$$